The use of conversation analysis in identifying creative approaches to mathematical problem solving

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Techniques of conversation analysis have been used in an effort to better understand the thought processes of adults engaged in a range of mathematical tasks. Participants are asked to provide a commentary during problem solving, in a non-judgmental environment with minimum intervention from the researcher.

Interesting outcomes from the work are: an inability to link arithmetic and algebra in problem solving, a lack of specialised mathematical vocabulary, misuse of standard algorithms which have been learned in a superficial manner without full understanding, and a preference for justification by concrete example rather than through abstract reasoning. Distinct differences in approach to problem solving are observed between participants with different preferred learning styles.

Keywords: Conversation analysis, adult numeracy, preferred learning styles.

Introduction

This investigation came about through a desire to better understand the problems faced by adults in improving their numeracy.

A first step when new students join a numeracy class is often to undertake an assessment of their mathematical skills through a written test. However, assessment of written answers may give limited insight into the thought processes of students. Where solutions are incorrect, this might variously be a result of: misinterpretation of the problem, lack of knowledge and understanding of solution methods, or inaccuracy in applying formulae and algorithms.

In this small project, six adults living in a town in North Wales and ranging in age from 20 to 60 were randomly selected as participants. None was in employment requiring specialised use of mathematics or numeracy beyond Adult Numeracy level 2, and none had undertaken any formal study of mathematics or numeracy since leaving school. Occupations included: office worker, computer technician and school teacher, and one participant had been unemployed for approximately a year. Although a limited sample, the group appeared to be fairly typical of the adult population as a whole.

Techniques of conversation analysis were used in an effort to better understand the thought processes of the participants (Ginsburg 1981). Subjects were asked to provide a commentary whilst engaged in a range of mathematical tasks, in a non-judgemental environment with minimum intervention from the researcher.

Investigating the arithmetic-algebra connection

An initial objective was to investigate the extent to which participants were able to make connections between arithmetic and algebra by substituting arithmetical values where appropriate to clarify algebraic expressions, and by formulating simple algebraic expressions to help in the solution of arithmetic problems (Lee and Wheeler 1989).

Task 1

Researcher: Would you look at this expression and say whether it is definitely true, definitely not true, or possibly true:

$$\frac{2x + 1}{2x + 1 + 7} = \frac{1}{8}$$

Participant: I would say it is not true... no, it is true, but it would have to be naught. That is my quick answer.

Four of the subjects spotted that the first expression was true for x = 0, but all then assumed that the value of x had to be zero so the expression was always true.

One participant commented: 'If x could have any value, then there are millions of answers and there is no way of checking if it's true. '

Task 2

Researcher: Could you do the same with this expression... say whether it is definitely true, definitely not true, or possibly true:

$$\frac{1}{6x} - \frac{1}{3x} = \frac{1}{3x}$$

Participant: I would say that would be definitely true because the sum of those two would be the other one.

Four participants made the error:

$$1/_6 - 1/_3 = 1/_3$$

No-one attempted to substitute numerical values to test the equality, relying instead on first impressions of the pattern of the equation.

Task 3

Researcher: Now try this question. Add and subtract numbers from 10, and see if the final total would always be the same for different starting values.
Participant: I would have to try this out with a range of numbers. If you use 7 you get 17... Then take away 7 is 3... So it comes to 20.
Let me go for 5. That is 15... and 5... and it's 20 again...
Well, yes, I think so. It would be the same.

The general approach was to try out several examples. The number of values chosen before deciding on the truth of the statement varied between two and four. Only one participant produced an algebraic expression:

$$(10+ N) + (10-N) = 20$$

as a proof of the assertion.

The outcome of tasks 1-3 suggested that the group of adults saw little connection between algebra and arithmetic, treating these as two entirely separate compartments of mathematics which could not be used together in any meaningful way for problem solving.

Using real objects

It has been suggested by Nunes, Light and Mason (1993) that the direct use of concrete objects in solving numeracy problems is less intellectually demanding than the use of mathematical methods, so will generally be preferred. To test this hypothesis, several tasks were devised using everyday physical objects. An example is given below:

Task 4

Researcher: I am going to show you this plastic lid and this tin. Could you say how you would work out whether the area of the plastic lid is bigger or smaller than the area of the paper label - without removing the label?

Participant: If I was to start with that line on the label at the edge of this sheet of plastic, and roll it very carefully like that. See where it goes...
I can see that it comes over the edge.
Whether it's got a smaller area –
It's got a smaller width, hasn't it.
So, as an estimate, although it goes longer here, its actually shorter there, so it's similar I would guess. About an inch wider here, and it seems about an inch shorter.

Researcher: If you wanted to be a bit more precise, how could you make some measurements?

Participant: I would roll the tin, starting with this edge on the label here, and see where it stops. If I can use a ruler... Then compare the two.

Researcher: Could you just imagine the label being unwrapped, so the circumference of the tin is the length of that rectangle.

Participant: Well, yes. So that would become a rectangle, and you could find the area of the rectangle.

Researcher: Would any of these formulae be true?

area of label = height of tin x circumference
area = π x radius^2 x height
area = π x height x diameter

Participant: Area of the label.... Height of the tin times circumference...
Yes, that's true, isn't it.
Area is pi r squared h… No, it's 2 pi r… I'm not sure.

When undertaking tasks involving real objects, the participants almost always attempted to solve the problem by physical measurements alone, without recourse to mathematical reasoning. When mathematical methods were suggested, there was an evident lack of recall of geometrical and algebraic techniques.

Investigating approaches to problem solving

It was felt that different individuals might have different approaches to problem solving, and some insight might be gained from assessing their preferred learning style. A variety of taxonomies of learning style have been proposed, but that of Roger Felder (1993) was chosen. Participants were evaluated using a questionnaire instrument to determine their positioning in respect to four dichotomies:

- How does the subject prefer to process information:
  actively—through engagement in physical activity or discussion, or
  reflectively—through introspection?
- How does the subject progress toward understanding:
  sequentially—in a logical progression of small incremental steps, or
  globally—in large jumps, holistically?
- What type of information does the subject preferentially perceive:
  sensory— sights, sounds, physical sensations, or
  intuitive— memories, ideas, insights?
- How is sensory information most effectively perceived by the subject:
  visually—pictures, diagrams, graphs, demonstrations, or
  verbally—sounds, written and spoken words and formulas?

Testing revealed a range of personal profiles for the six participants, indicated on a scale from 1 (low preference) to 11 (high preference) within each dichotomy:

Subject	Sensing	Intuitive	Visual	Verbal	Sequential	Global	Active	Reflective
1	2	9	9	2	3	8	2	9
2	4	7	6	5	7	4	7	4
3	9	2	11	0	5	6	6	5
4	11	0	8	3	8	3	6	5
5	10	1	2	9	6	5	2	9
6	2	9	4	7	6	7	5	6

Moderate preference   Strong preference

Participants were then asked to undertake a series of tasks, and an attempt was made to relate their approach to problem solving to the assessed learning style preferences. Examples are presented below:

Task 5

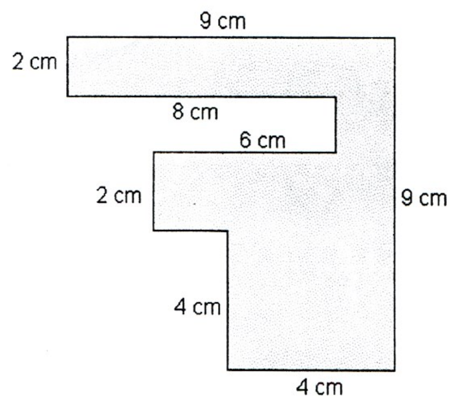
Researcher: Could you read this problem and have a go at solving it:

Participant: The teachers at Cwm Coed School decide to organise a fund to pay for gifts to teachers who leave the school.
They decide to pay £5 a year into this fund. When a teacher leaves, he or she is given £30 plus £3 for every year the teacher has been at the school.
After how many years at the school would then amount paid in by the teacher equal the amount of the gift received on leaving?

Well, if they stayed for six years they would get £30 plus ... they would also get £3 for every year they had been teaching. Ur... for six years that would be £48.
(pause … 6 seconds)
The difference between 3 and 5 is £2. Uhm...
In 15 years they would get £75 and pay £75. I see it, yes! So it's 15 years.

The participant (subject 6) worked entirely ***verbally*** throughout, writing nothing on paper. There was no recourse to any algebraic technique. The final solution was reached in a moment of ***intuition***. This fitted remarkably with their preferred learning style as strongly intuitive and moderately verbal.

Task 6



Researcher: This is a shape, and I would like you to work out the distance round the edge of it.

Participant: The distance round the edge of it...
(begins counting the distances by mental arithmetic, then stops)
We haven't got how long that is...
We could take that from that... no, that wouldn't work...
It's not as straight forward as I first thought. Oh dear...

(pause … 11 seconds)

Researcher: Can we do anything with those two measurements?

Participant: Oh, yes. So that's one... That's six....It will work this time..
So that's three... and that's one as well...
(using calculator)……. 48
Sorry, that took a long time!

The participant's difficulty in seeing the solution to this apparently simple geometrical problem may lie in very strong preference for ***sensing*** and ***sequential*** learning styles (subject 4), as opposed to the ***intuitive*** and ***global*** approaches which might have been more successful.

Conclusions

This study is ongoing, but has begun to provide interesting insight into adult numeracy. The conversation extracts above are part of a larger body of data which allows some general conclusions to be drawn:

Within a few years of leaving school, and with no further formal study of mathematics or numeracy, adults lose much of their familiarity with algebra and geometry. They are hampered by a lack of specialised mathematical vocabulary when exploring the solution of problems, and frequently misuse standard algorithms which have been learned in a superficial manner without full understanding.

A particular difficulty arises from an inability to link arithmetic and algebra in meaningful ways during problem solving. There is a strong preference for justification by concrete example and direct measurement, rather than through abstract reasoning. This is particularly evident when problems are presented which involve physical objects.

Distinct differences in approach to problem solving are observed between participants with different preferred learning styles. Individuals appear to develop their own unique mathematical coping strategies which may diverge widely from the standard methods taught in schools. This seems an interesting area for further investigation.

It became apparent during the research that there is a tendency for the teacher to intervene too quickly when a response is not forthcoming, allowing insufficient time for reflective learners to think through the problem and develop their own, perhaps unique, solutions.

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