## 2.3 Meteorological modelling

The derivation of mathematical models for meteorology is presented by Jacobson (1999) and Grell, Dudhia and Stauffer (1995).

Meteorological models may cover the whole globe for world climate modelling, or may be restricted to particular areas for weather forecasting or research. Models focussing on particular areas are known as *mesoscale models*. A mesoscale meteorological model uses an outer domain which links to the global weather circulatory pattern, and one or more high resolution inner domains covering areas of special interest (fig.2.61).

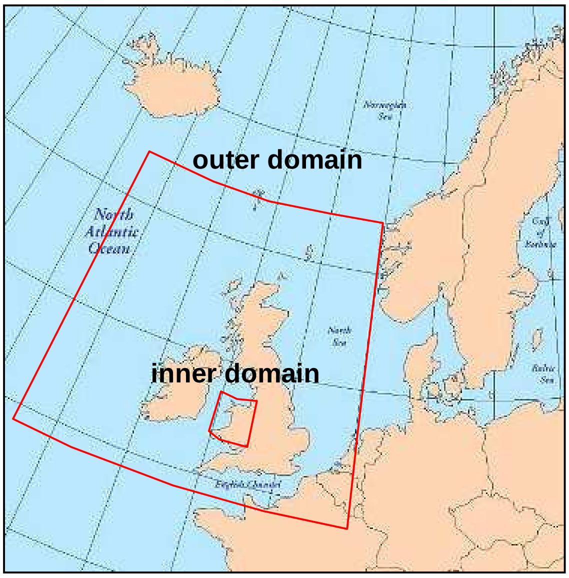



**Figure 2.61: Nested mesoscale domains**

The outer domains of mesoscale models often extend for more than 1000km, so the curvature of the Earth cannot be ignored when calculating distances. Models are usually represented by the Spherical Polar Coordinate system (fig.2.62).

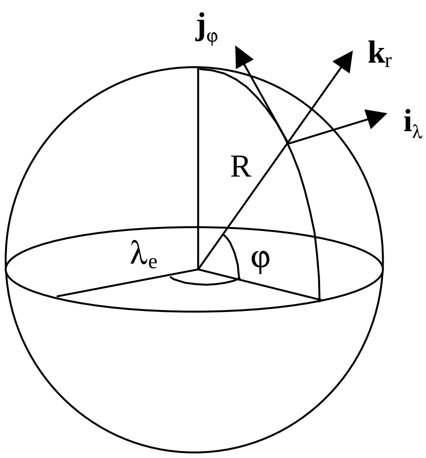



**Figure 2.62: Spherical Polar Coordinates**

The spherical polar system uses lambda ($\lambda_e$) to represent longitude, and phi ($\varphi$) to represent latitude. The radius of the Earth ($R_e$) is also required for the calculation of distances on the Earth's surface.

The inner domain of a mesoscale model can operate with a horizontal grid spacing of 1km or less, and will take into account local factors such as topography, soil types and vegetation cover which can influence surface air temperatures and wind vectors.

The model must represent processes taking place in the lower part of the atmosphere, the *troposphere*, up to an altitude of around 10km and air pressure of 250mb. This region would typically be modelled by between 20 and 30 atmospheric layers. Problems can arise if horizontal layers are employed, as these may intersect the ground surface topography and increase the complexity of calculations. An alternative approach using ground-following coordinates is generally used, in which each layer boundary represents a fixed fraction of the pressure difference between the ground surface and the top surface of the model. This is known as the sigma-pressure ($\sigma$) coordinate system (fig.2.63).

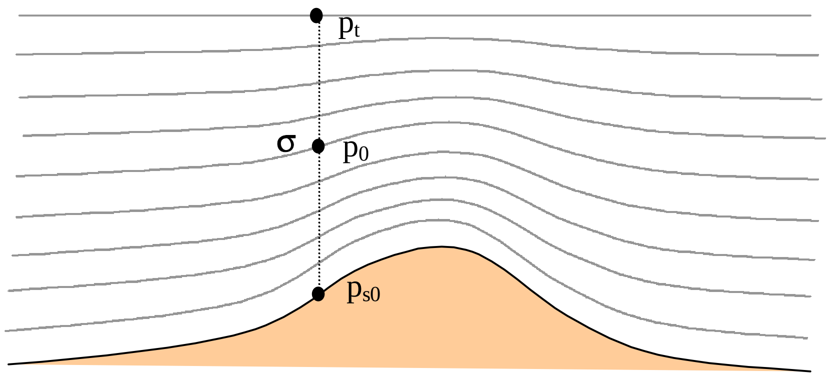



**Figure 2.63: Sigma-pressure coordinate system**

The sigma value at any point may be calculated using the relation:

$$\sigma = \frac{(p_0 - p_t)}{(p_{s0} - p_t)}$$

| | |
|---|---|
| $p_0$ | local pressure |
| $p_{s0}$ | ground surface pressure |
| $p_t$ | model top surface pressure |

A complexity of meteorological models in comparison to hydrological models is that air is compressible, whereas water is normally considered to be incompressible.  Dry air behaves as a close approximation to the Ideal Gas Law:

$$p = \frac{nRT}{V}$$

where:

| | |
|---|---|
| p | pressure |
| n | moles of gas |
| R | universal gas constant |
| T | absolute temperature |

Pressure naturally varies with altitude.  An air parcel at any point in the atmosphere will be subject to the weight of the overlying column of air, which increases towards the ground surface (fig.2.64).

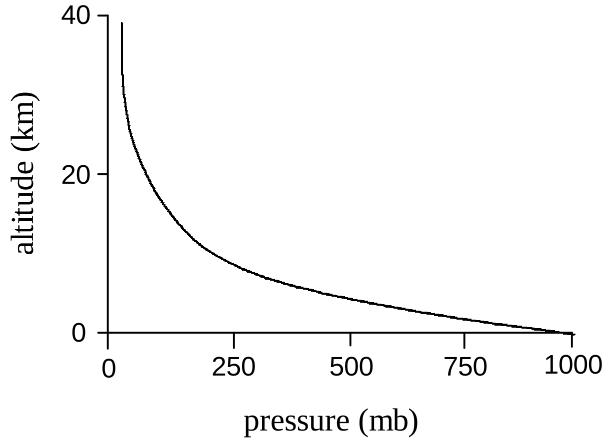



**Figure 2.64:  Pressure distribution in the lower atmosphere**

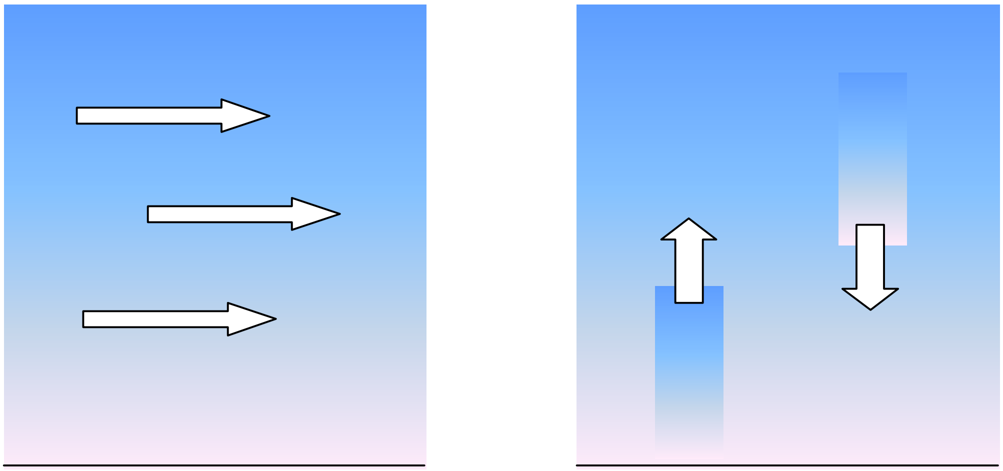
The relationship between altitude and pressure is given by the hydrostatic equation:

$$\frac{\partial p}{\partial z} = -\rho g$$

where:

| | |
|---|---|
| g | acceleration due to gravity |
| ρ | density of air |
| p | air pressure |
| z | altitude |

Meteorological models covering large areas at low resolution often make a simplifying assumption that the atmosphere remains in vertical equilibrium according to the hydrostatic equation.  This assumption is realistic provided that horizontal air motions predominate (fig.2.65a).



(a) hydrostatic model (b) non-hydrostatic model

**Figure 2.65: Pressure distributions in meteorological models**

For high resolution models of small areas, vertical air motions due to convection may be important. Heating or cooling of air parcels may lead to pressure and density variance from the surrounding stable atmosphere. In this case, the model must allow for non-hydrostatic pressure distributions (fig.2.65b).

A fundamental principle of atmospheric models is the conservation of air mass, described by the differential equation:

$$\frac{\partial N}{\partial t} = -\frac{\partial (uN)}{\partial x} - \frac{\partial (vN)}{\partial y} - \frac{\partial (wN)}{\partial z}$$

where N is the mass of air; u,v and w are velocities in the cartesian directions x,y and z; t is time.

Meteorological model development begins with the equations of atmospheric physics:

- hydrostatic equation
- continuity equation
- thermodynamic energy equation
- momentum equation

which form the basis for a simple dry air model. Jacobson (1999) provides an outline modelling scheme (fig.2.66). These equations are discussed further in section 2.4.

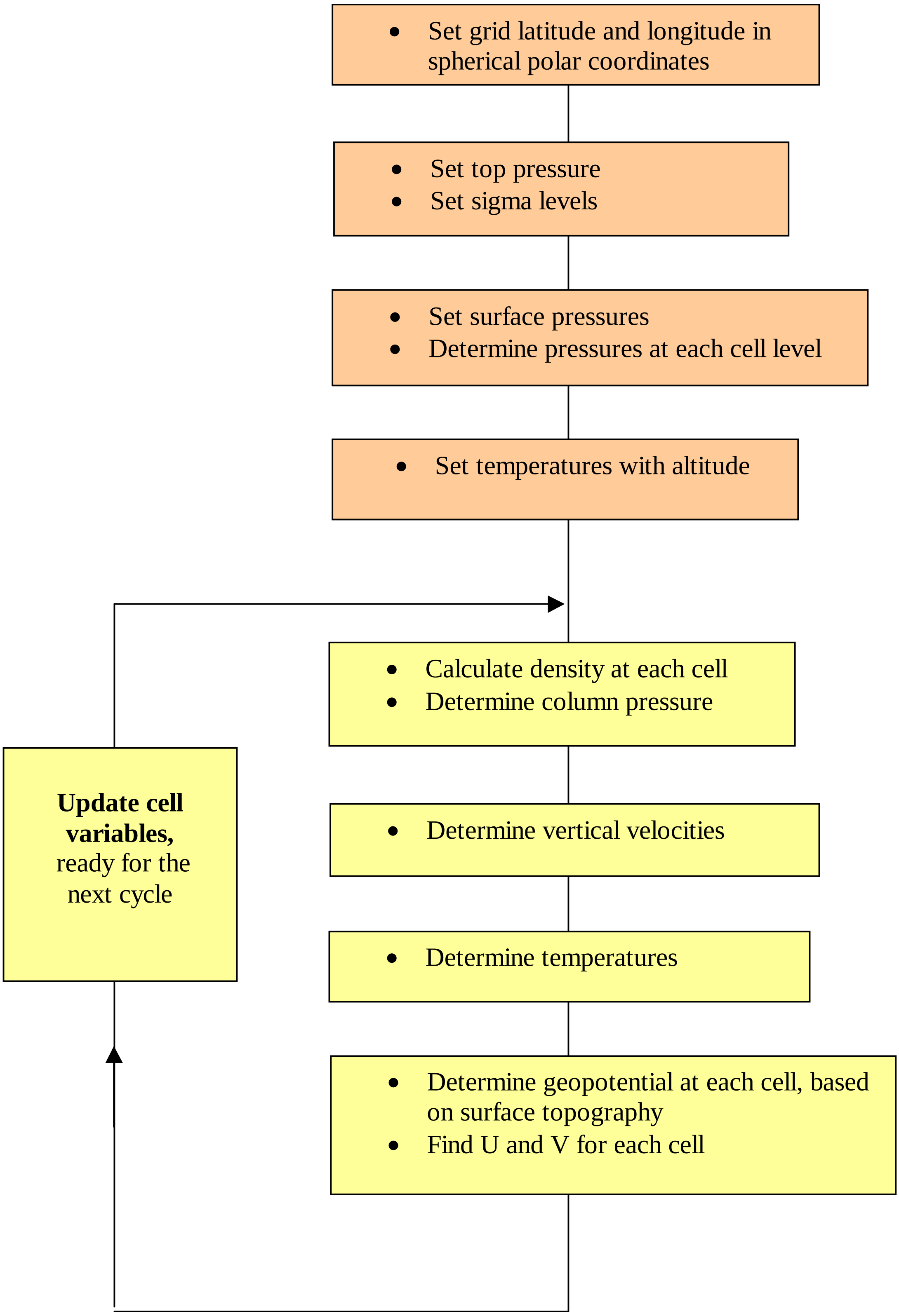



**Figure 2.66: Jacobson scheme for a simple dry air meteorological model**

The model is initialised by setting up a horizontal grid to cover the desired area, bounded by lines of latitude and longitude on the Earth’s surface. The top atmospheric level for the model is chosen – typically this would be at 10km, and sigma levels are set between the top level and the ground surface topography.

Initial air pressures for the boundary cells of the grid are specified, with the top level of the model having a constant pressure of around 250mb throughout the simulation. Pressures are interpolated to each model cell for the start of the run. Temperatures of boundary cells are similarly initialised and interpolated to the interior cells.

The main loop of the model, shown in yellow in fig.2.66, repeats for each time interval during the simulation:

At the start of a time step, the air densities for each cell are determined, and used to calculate pressures in each cell according to the overlying air column mass.

Vertical air velocities are calculated for each cell, depending on pressure deviation from hydrostatic equilibrium and air viscosity forces. Air temperatures are determined for each cell, according to the Ideal Gas Law.

The geopotential for each cell is calculated. This is a measure of the gravitational potential energy of the air within the cell, and varies with altitude. This factor must be considered when sigma levels are used, since the altitude of a sigma layer boundary may vary across the model area according to the underlying topography.

It is now possible to calculate the horizontal velocity vectors for air within the cell, taking into account gravity, pressure gradient, Coriolis and viscosity forces. Pressure, temperature, horizontal and vertical velocity values can be updated for each model cell, and the cycle repeated for the next time interval.

Anthes and Warner (1978) develop a mathematical treatment of the Earth's physical boundary layer, and the modelling of precipitation by convective cloud processes. These aspects will be discussed further in section 2.4 in the context of the MM5 meteorological model.

## Using atmospheric soundings

Rainfall production is generally the result of cooling and condensation of water vapour as air rises. Water droplets may initially form from saturated air in contact with airborn particles such as dust, smoke or salt crystals, then grow by accretion within a cloud (Barry and Chorley, 1976). Field evidence of air conditions and the likelihood of cloud formation can be obtained from the ascents of weather balloons through the troposphere. Instruments are carried which measure air temperature and moisture content, relaying this information back to the ground station along with the balloon altitude and geographical location. From this data, it is also possible to determine wind speed and direction at different altitudes during the ascent.

Data collected from balloon ascents, termed *soundings*, are generally plotted on pressure-temperature charts termed *tephigrams,* named from the mathematical symbols T used for temperature and ϕ(phi) used for pressure. An example is given in fig.2.67 and its interpretation is explained below. Atmospheric soundings provide

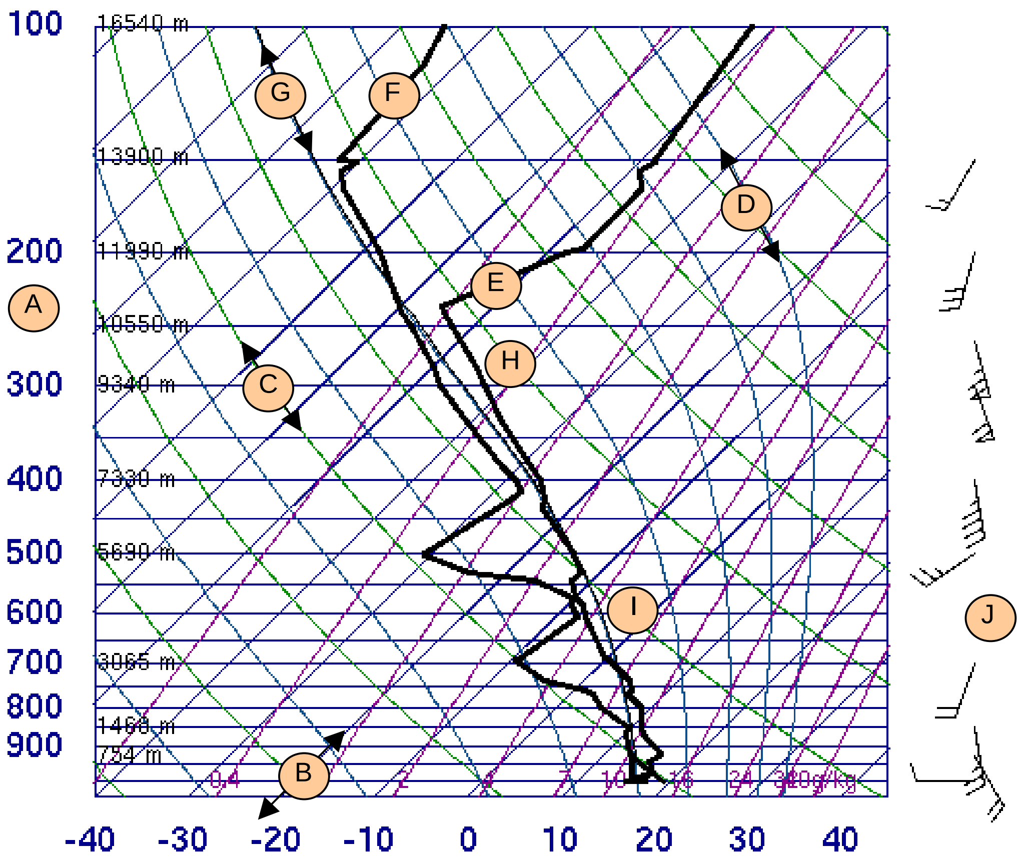



**Figure 2.67: An example Tephigram**

input data for the meteorological model described in Section 2.4. After a run of the model, synthetic tephigrams can be plotted for a sequence of locations or times to demonstrate the atmospheric conditions produced during the simulation.

Key to features of the tephigram:

A: The vertical axis represents altitude, shown as pressure (mb) and height (m) scales.

B: The horizontal axis represents temperature in degrees Celsius. Lines of constant temperature are shown in blue, running upwards towards to right. For this reason, tephigrams are also known as *skew-T* plots.

C: Green lines curving upwards to the left, termed *dry adiabats*, represent the pressure-temperature paths which would be followed by parcels of dry air rising through the atmosphere.

D: More steeply curving blue lines, termed *moist adiabats*, represent the pressure-temperature paths which would be followed by parcels of saturated air from which water vapour was condensing during rise through the atmosphere.

E: The right-hand thick graph line represents the dry bulb air temperatures recorded during the balloon ascent.

F: The left-hand thick graph line represents the wet bulb air temperatures recorded during the ascent. The horizontal distance between the two graph lines relates to the saturation state of the air. Where the lines are close together, the air is near saturation and condensation, cloud formation and rainfall are likely. Where the lines are far apart, the air is drier and condensation is unlikely.

G: This curving black line represents the ascent path that a parcel of air would follow if it began to rise from the ground at the point of release of the balloon. This is known as the *parcel lapse rate*. The path line initially follows a dry adiabat curve to the point where condensation begins, then follows a saturated adiabat for the remainder of the ascent.

H:  The relative positions of the parcel lapse rate line and the dry bulb temperature line give information on the stability of the air.  If the parcel lapse rate line is furthest to the left, as at the level of H, then  the air is stable.  A rising parcel will be colder and denser than the surrounding air, so has no tendency to continue its upward motion.

I:  Where the dry bulb temperature line is to the left of the parcel lapse rate line, as at the level of I, the air is unstable.  A rising parcel will be warmer and less dense than the surrounding air, so continues to rise buoyantly.

J:  Symbols to the right of the graph represent the compass orientation of winds at different levels during the balloon ascent.  Barbs on the arrows are an indication of wind speed.

Balloon ascent data is valuable for understanding atmospheric conditions and relating these to weather patterns.  The use of atmospheric data can be demonstrated for an example rainfall event of 24 October 2005:

Fig.2.68 shows rainfall distributions for 00:00h and 12:00h on 24 October 2005.  A rain band moves across Ireland and the mainland of Britain as a warm front advances. Uplift to the NW along a warm air conveyor is responsible for the precipitation.  Data from radiosonde balloon ascents at 00:00h and 12:00h are available for the stations Camborne, Castor Bay and Nottingham.

Data for 00:00h are plotted as tephigrams in fig.2.69.  The closeness of the dry bulb and wet bulb lines up to 3 000m is a good indicator of rainfall occurring.  Air at Castor Bay and Nottingham is stable, with the dry bulb temperature graph to the right of the parcel lapse rate curve for much of the ascent.  Removal of air by cyclonic vorticity is the driving force for ascent, rather than natural buoyancy of the air. Marked changes in wind direction within the lower 3 000m for these stations is an indicator of the vorticity.

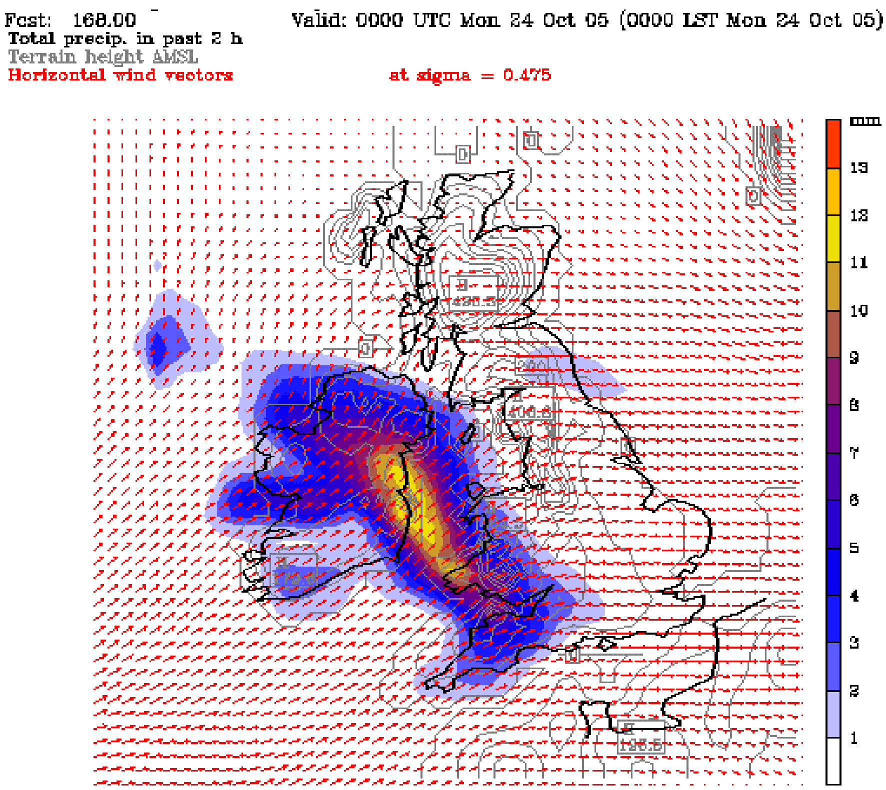



**Figure 2.68(a). Modelled two-hour rainfall total for 00:00h, 24 October 2005**

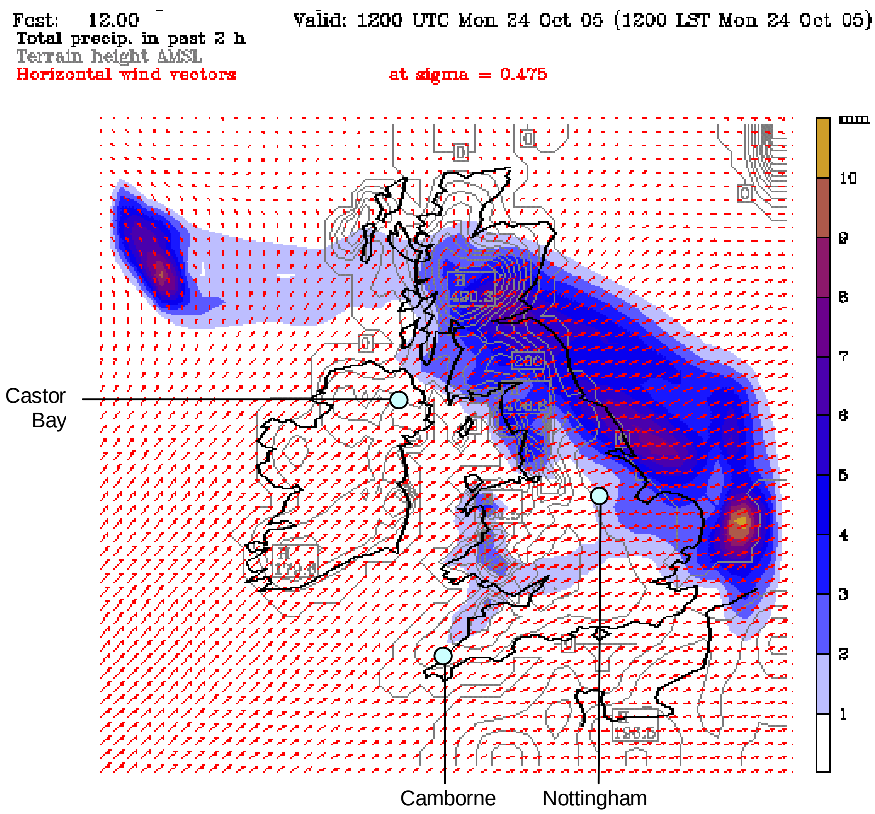



**Figure 2.68(b). Modelled two-hour rainfall total for 12:00h, 24 October 2005**

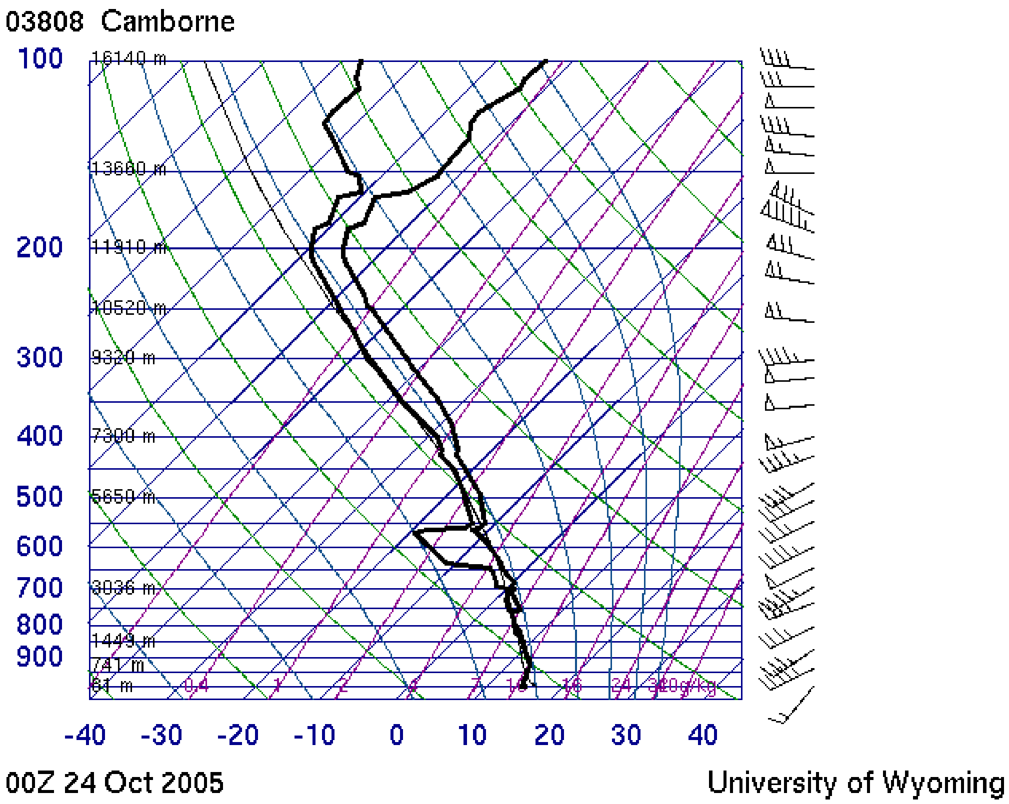



**Figure 2.69(a). Actual atmospheric profile for Camborne, 00:00h 24 October 2006**

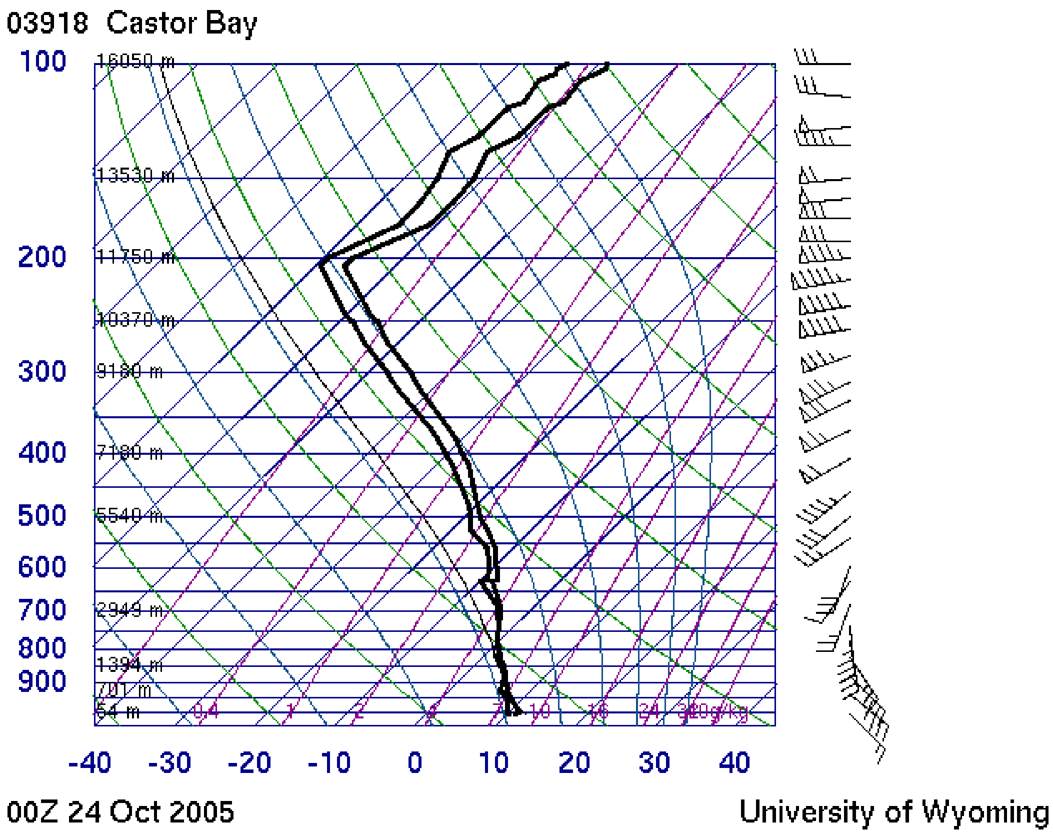



**Figure 2.69(b). Actual atmospheric profile for Castor Bay, 00:00h 24 October 2006**

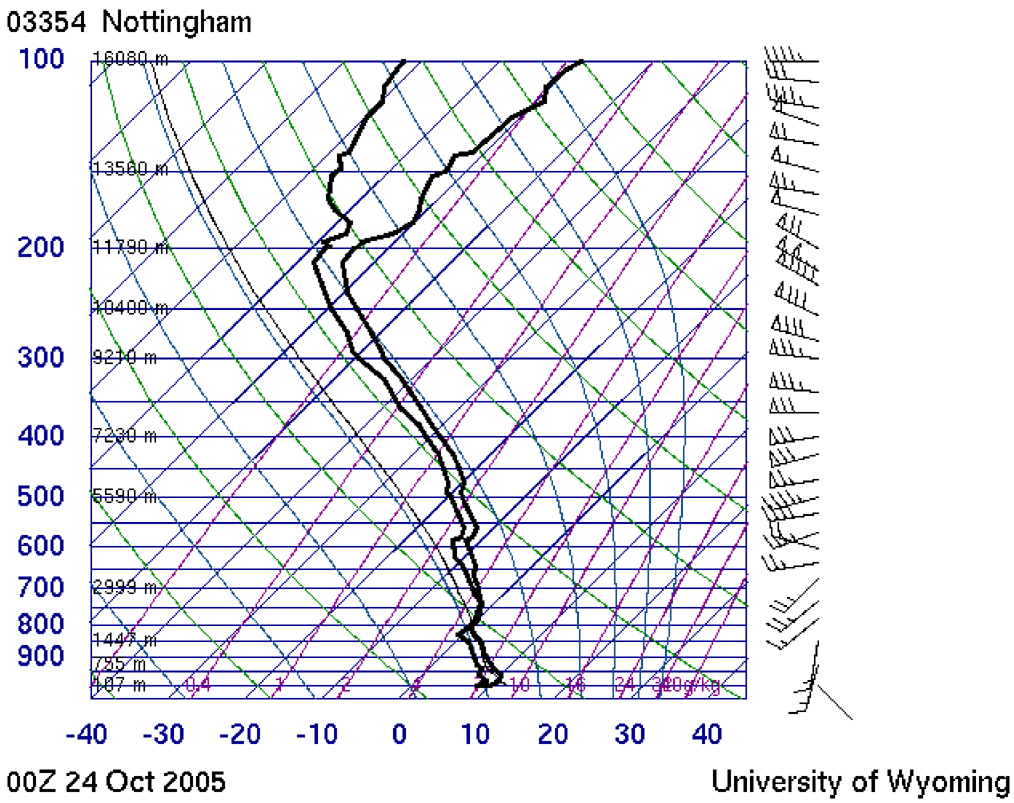



**Figure 2.69(c). Actual atmospheric profile for Nottingham, 00:00h 24 October 2006**

This example of frontal rainfall may be contrasted with the squall line convective event responsible for flooding in the Mawddach catchment in July 2001.

Tephigrams for Camborne over the period 12:00h, 3 July 2001, to 00:00h, 4 July 2001, are presented in figure 2.70. The profiles show strong instability at altitudes between 3000m and 5000m for the afternoon and evening of 3 July, with the dry bulb trace lying to the left of the parcel lapse rate curve. Features of instability have been highlighted in red on the graphs.

Statistical information about the state of the atmosphere is computed during the analysis of balloon ascent data, and is shown to the right of the tephigram. Of particular interest in this case is the amount of *convective available potential energy* (CAPE) recorded from the profile, which is a measure of the tendency of the air to

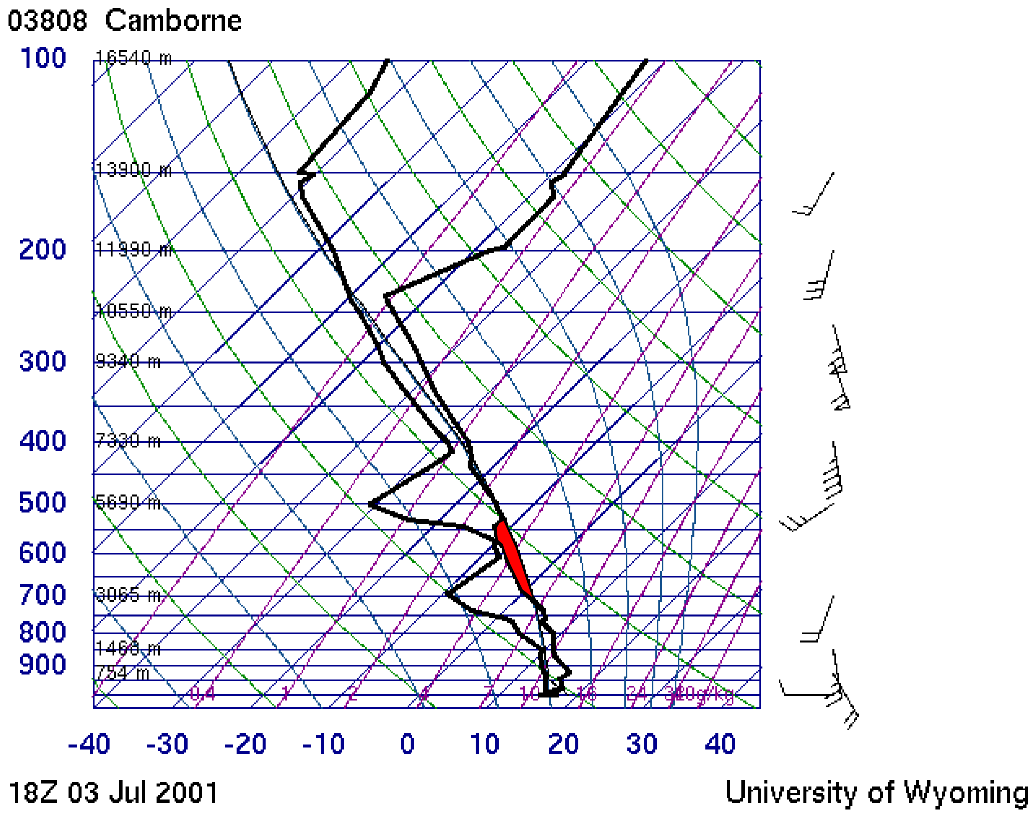



**Figure 2.70(a). Atmospheric profile for Camborne, 18:00h 3 July 2001**

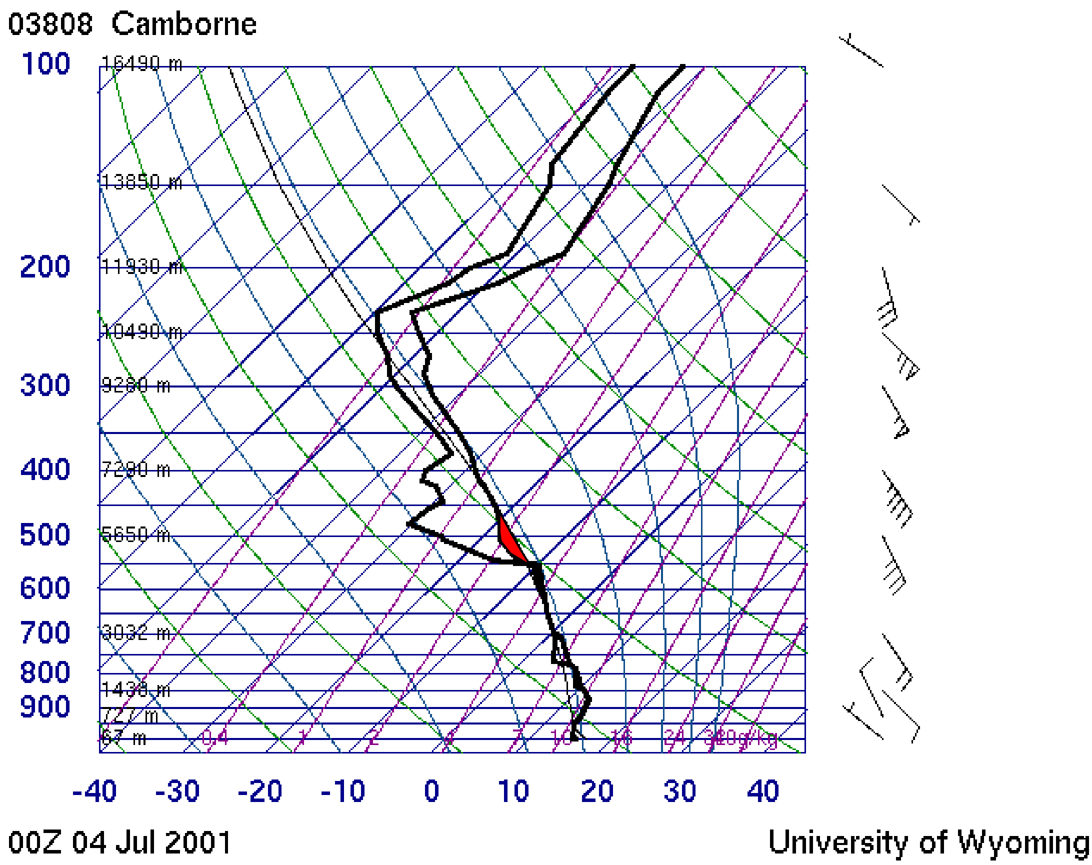



**Figure 2.70(b). Atmospheric profile for Camborne, 00:00h 4 July 2001**

undergo upwards convective motion. The appearance of any positive value for CAPE is an indicator that thunderstorm activity is possible.

Atmospheric soundings are of considerable value in providing data for calibration of meteorological models. They can provide important data on atmospheric motion and moisture content which may in turn be used in calculating the location, timing and intensity of rainfall events.

**Summary**

- Meteorological modelling may be carried out on a variety of scales. Models focussing on particular regions are known as mesoscale models.
- Models often consist of a set of nested domains, with each providing boundary conditions for a higher resolution domain which it encloses.
- Meteorological models usually cover a sufficient area that the curvature of the Earth must be taken into account when calculating horizontal coordinates.
- The sigma-coordinate system has been devised to simplify the modelling of atmospheric layers above undulating surface topography. Sigma levels represent specified fractions of the total pressure interval between the ground and the upper model surface at an altitude of typically 10km.
- Simple models may make an approximation of hydrostatic pressure distribution upwards through the atmosphere. More accurate models incorporating convection processes must allow for a non-hydrostatic pressure distribution.
- Models are initialised with temperature and pressure values at a series of sigma levels for a grid of points covering the outer domain.
- The model is run iteratively to generate horizontal and vertical air velocities, pressures and temperatures for each grid point over a series of time steps.
- Values for moisture content are added for each cell, and processes of condensation and precipitation can be simulated to produce rainfall forecasts.
- Temperature, pressure and moisture content data for initialising and updating the model may be obtained from balloon soundings in addition to ground stations.