Equation of a straight line

Learning points

The equation for a straight line graph is:

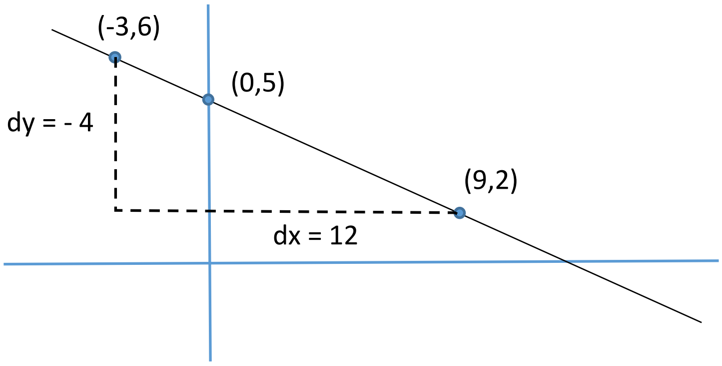
$$y = mx + c$$

where m is the gradient of the line, and c is the point at which the line cuts the vertical y-axis.

The gradient of the line is defined as:

$$m = \frac{dy}{dx} \quad , \qquad or \quad \frac{change\ in\ y}{change\ in\ x}$$

If the line rises as x increases, then the gradient is positive. If the line falls as x increases, the gradient is negative. For example, for a line passing through the points (-3,6) and (9,2):



$$m = \frac{-4}{12} = -\frac{1}{3}$$

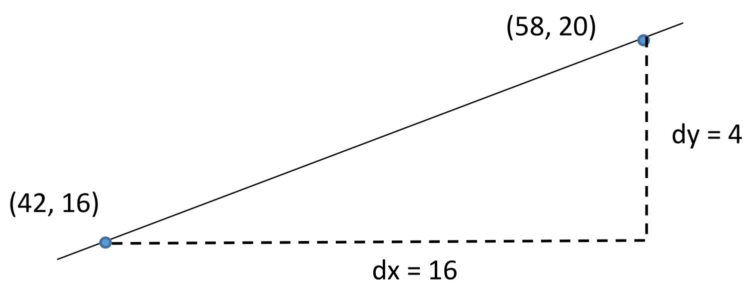
If the line cuts the y-axis at the point (0,5) then the full equation of the line is:

$$y = -\frac{1}{3}x + 5$$

An alternative equation for a straight line is:

$$y - y_1 = m(x - x_1)$$

In this expression, (x_1, y_1) is any known point on the line, and m is again the gradient. It can be convenient to use this equation if the coordinates of the point where the line cuts the y-axis are not known. As an example, we will find the equation of a line passing through the points (42, 16) and (58, 20). A first step is to calculate the gradient:



$$m = \frac{4}{16} = \frac{1}{4}$$

Either of the known points can now be substituted in the equation. Taking the point (42, 16):

$$y - y_1 = m(x - x_1):$$

$$y - 16 = \frac{1}{4}(x) - \frac{1}{4}\,(42)$$

giving:

$$y = \frac{1}{4}x + 5\frac{1}{2}$$

Alternatively, we could have taken the point (58, 20):

$$y - 20 = \frac{1}{4}(x) - \frac{1}{4}\,(58)$$

giving the same result of:

$$y = \frac{1}{4}x + 5\frac{1}{2}$$

Another form which can be used as the equation of a straight line is:

$$ax + by + c = 0$$

This simply requires the rearrangement of the standard form $y = mx + c$

For example, using: $y = -\frac{1}{3}x + 5$, then:

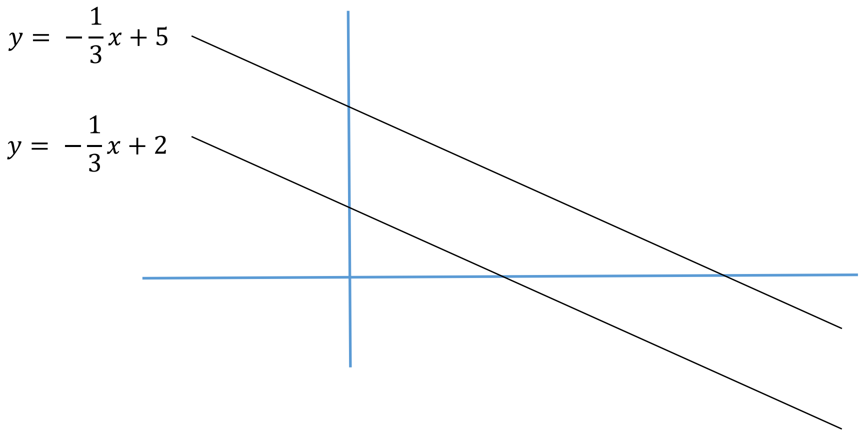
$$\frac{1}{3}x + y - 5 = 0\,, \quad or\ \ x + 3y - 15 = 0$$

Parallel lines

Where two lines are parallel, they must have the same gradient. Using our first example:

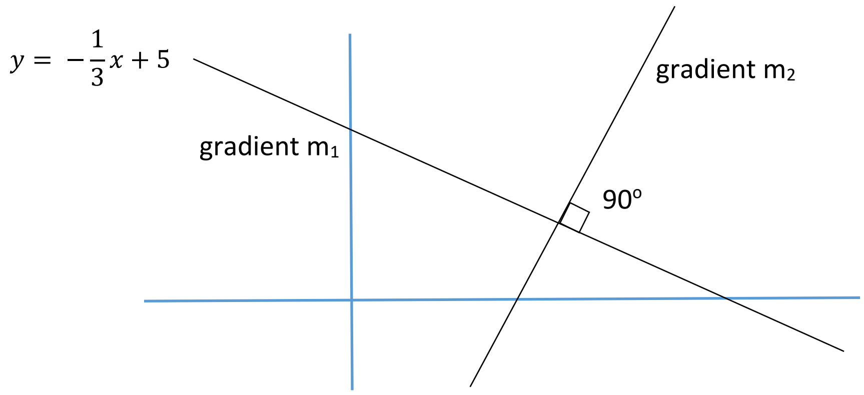
$$y = -\tfrac{1}{3}x + 5$$

then any other line with a gradient of $-\frac{1}{3}$ would be parallel:



Perpendicular lines

Lines which are perpendicular lie at right angles to one another:



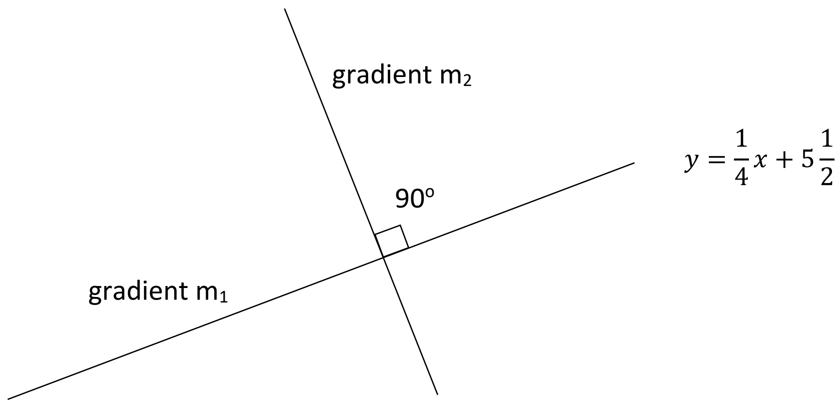
Lines are perpendicular if their gradients are related by the equation:

$$m_2 = -\frac{1}{m_1}$$

A line perpendicular to $y = -\frac{1}{3}x + 5$ must have a gradient

$$m_2 = -\frac{1}{\left(-\frac{1}{3}\right)} = 3$$

In the case of the line $y = \frac{1}{4}x + 5\frac{1}{2}$,

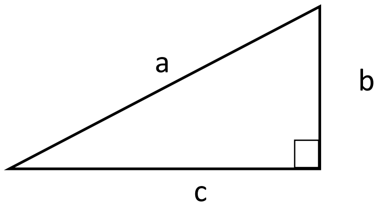


any perpendicular line will have a gradient:

$$m_2 = -\frac{1}{\left(\frac{1}{4}\right)} = -4$$

Finding the length of a line

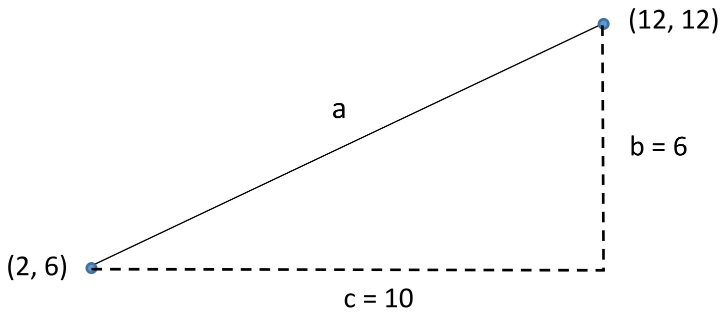
If the cooordinates of the end points of a line are known. then its length can be calculated using Pythagoras' theorem for the sides of a right-angled triangle:



$$a^2 = b^2 + c^2$$

where a is the length of the hypotenuse, and b and c are the lengths of the other two sides.

Consider a line between the points (2,6) and (12, 12):



$$a^2 = 6^2 + 10^2$$

$$a = \sqrt{136} \quad = \quad 11.66$$

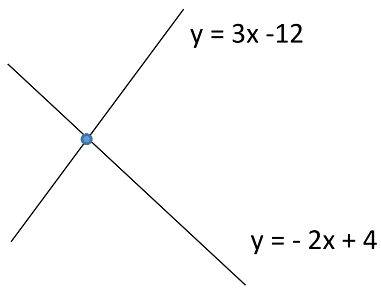
Intersection of straight lines

Consider two straight lines with the equations:

$$y = 3x - 12$$

$$y = -2x + 4$$

It is required to find the point at which the lines intersect:



This problem can be solved by combining the two equations. At the intersection point, the y-values will be equal, so:

$$3x - 12 = -2x + 4$$

$$5x = 16 \qquad so \qquad x = 3.2$$

Substituting for x in one of the original equations gives:

$$y = 3\left(\frac{16}{5}\right) - 12$$

$$y = -2.4$$

The lines therefore cross at the point (3.2, -2.4)

Practical applications

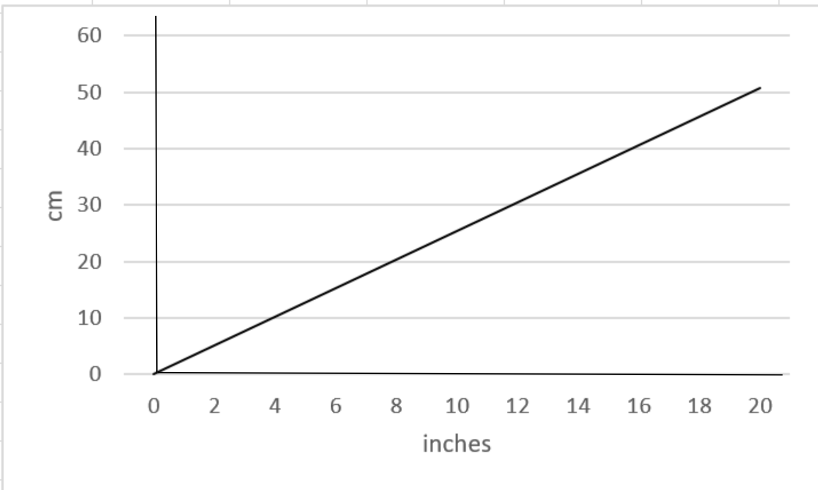
Linear graphs

Straight line graphs may be used for conversion between units in different measurement systems.

It is necessary to know the conversion factor. For example:

$$1\ inch = 2.54\ cm$$

A graph can then be plotted to provide a conversion chart:



The graph has the simple equation: $y = 2.54\ x$ where y is centimetres and x is inches.

Conversion of degrees Farenheit to degrees Celsius is a little more complicated:

- The freezing point of water is 32° Farenheit or 0° Celsius.
- The boiling point of water is 212° Fahrenheit or 100° Celsius.

The graph needs to pass through the points (32, 0) and (212, 100). Using the formula:

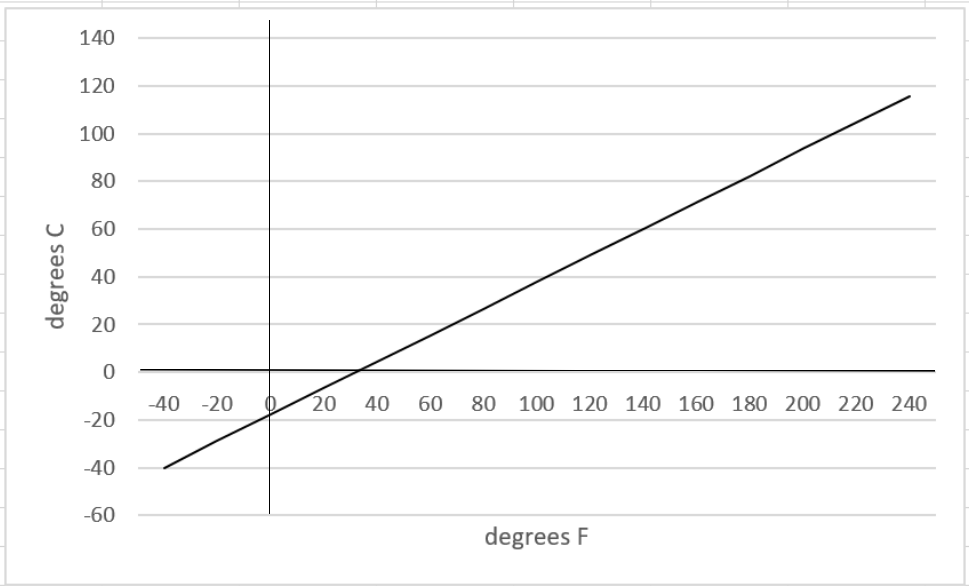
$$C - C_1 = m(F - F_1):$$

the gradient $m = \frac{change\ in\ C}{change\ in\ F} = \frac{100}{180} = \frac{5}{9}$

Substituting for the point (32, 0):

$$C - 0 = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}F - \frac{160}{9}$$



It is interesting to calculate the temperature at which the readings would be the same on both the Farenheit and Celsius scales. We can do this by setting C = F to give the equation:

$$F = \frac{5}{9}F - \frac{160}{9}$$

$$\frac{4}{9}F = -\frac{160}{9}$$

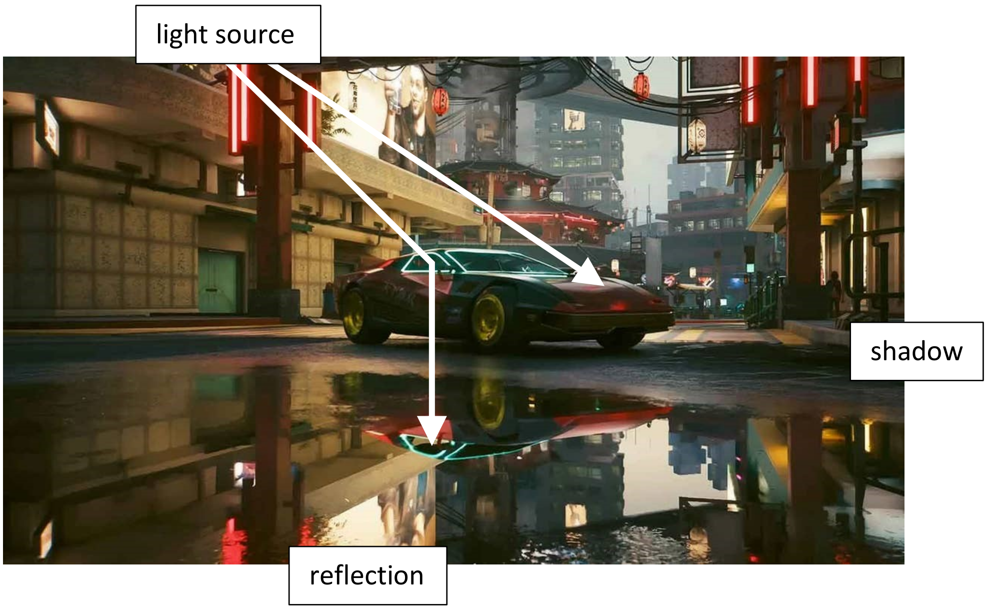
$$4F = -160 \quad or \quad F = -40$$

Thus, Farenheit and Celsius readings would be the same at – 40 degrees.

Computer graphics

Calculations involving straight lines are an essential component of many computer graphics applications, including three dimensional scenes for computer games, and computer aided design applications for engineering and achitecture.

A particular technique used in producing realistic computer generated images is ***ray tracing***. This involves tracing the straight line paths that rays of light would take. It allows the program to determine the colours that should be shown when the light reflects off shiny surfaces such as metal or water. Ray tracing also identifies where light is blocked by a solid object, producing a shadow.



Multiple choice questions

1.

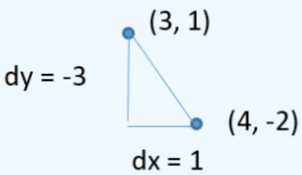
The line *l* passes through the points *A* (3, 1) and *B* (4, – 2).

Find an equation for *l*.

Select an answer:

A: $y = 3x + 10$ B: $y = -3x + 12$ C: $y = -3x + 10$ D: $y = -3x + 6$

Correct answer: C



$$gradient = \frac{dy}{dx} = -3$$

Considering the point (3,1) on the line:

$$y = mx + c, \quad so \quad 1 = -9 + c$$

$$c = 10$$

Hence, the line has the equation: $y = -3x + 10$

2.

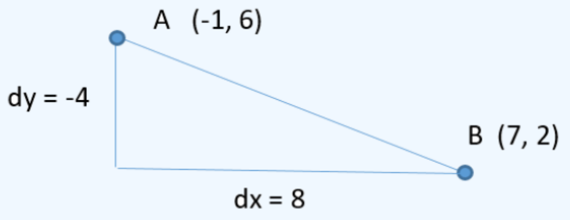
The point A has coordinates (-1, 6) and the point B has coordinates (7, 2)

Find the equation of the perpendicular bisector of AB.

Select an answer:

A: $y = -2x + 4$ B: $y = 2x - 2$ C: $y = 6x - 3$ D: $y = 2x + 2$

Correct answer: B



gradient = $-\frac{1}{2}$

The mid point of AB is given by: (mean x-value, mean y-value) = (3, 4)

Lines are perpendicular if the product of their gradients is -1. Thus:

$$gradient\ of\ perpendicular \times -\frac{1}{2} = -1$$

The gradient of the perpendicular to AB is 2.

Taking the mid-point of AB as a point on the perpendicular, then

$$y = mx + c \qquad so \qquad 4 = 2(3) + c$$

$c = -2$, giving the equation of the perpendicular as: $y = 2x - 2$

3.

The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular, find the value of m.

Select an answer:

A: 3 B: $\frac{3}{4}$ C: 2 D: $\frac{1}{2}$

Correct answer: C

Rearranging the equation of line l_1:

$$4y = -2x + 3 \qquad so \qquad y = -\frac{1}{2}x + \frac{3}{4}$$

Lines l_1 and l_2 are perpendicular if their gradients multiply to give -1. Hence:

$$-\frac{1}{2} \times m_2 = -1 \qquad so\ m_2 = 2$$

The formula for line l_2 is: $y = 2x + 7$

4.

Line L_1 has the equation $y = -3x + 18$. Line L_2 has the equation $y = \frac{1}{3}x + 8$.

Find the coordinates of the point where the lines cross.

Select an answer:

A: (-3, 9) B: (3, 9) C: (6, -3) D: (3, 6)

Correct answer: B

At the point where the lines cross, the y-coordinates on both lines will be equal. Therefore:

$$-3x + 18 = \frac{1}{3}x + 8$$

$$-\frac{10}{3}x = -10, \qquad so \qquad -10x = -30, \qquad x = 3$$

Substituting x = 3 in the equation for line L_1:

$$y = -3(3) + 18, \qquad so\ y = 9$$

The lines cross at the point (3, 9)

5.

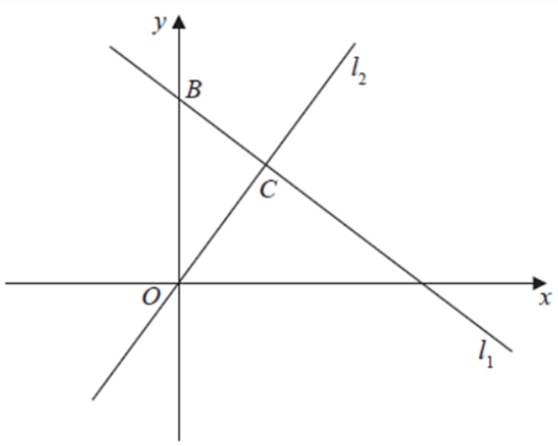


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

Find an equation for the line l_2

Select an answer:

A: $y = \frac{3}{2}x$ B: $y = -\frac{3}{2}x$ C: $y = \frac{2}{3}x$ D: $y = 3x$

Correct answer: A

Line l_1 has the equation:

$$2x + 3y = 26, \quad so \quad 3y = -2x + 26$$

$$y = -\frac{2}{3}x + \frac{26}{3}$$

Thus, the gradient of line l_1 is $-\frac{2}{3}$. If line l_2 is perpendicular to l_1, then:

$$gradient\ l_2 \times -\frac{2}{3} = -1$$

The gradient of l_2 is $\frac{3}{2}$. Its equation is: $y = \frac{3}{2}x + c$

Line l_2 passes through the point (0, 0). Substituting these values, c=0. Thus the equation of l_2 is:

$$y = \frac{3}{2}x$$

6.

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

Find an equation for L_1 in the form $ax + by + c = 0$,

where a, b and c are integers.

Select an answer:

A: $4x + 4y + 12 = 0$ B: $3x - 4y + 15 = 0$

C: $3x - 4y - 9 = 0$ D: $6x - 4y + 3 = 0$

Correct answer: B

The gradient of line L_1 is given by:

$$gradient = \frac{dy}{dx} = \frac{(12-3)}{(11-(-1))} = \frac{9}{12} = \frac{3}{4}$$

The equation of L_1 is therefore:

$$y = \frac{3}{4}x + c$$

The line passes through the point x = -1, y = 3. Substituting these values:

$$3 = \tfrac{3}{4}(-1) + c, \quad so\ c = 3\tfrac{3}{4}$$

$$y = \tfrac{3}{4}x + 3\tfrac{3}{4}, \quad so \quad 3x - 4y + 15 = 0$$

7.

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

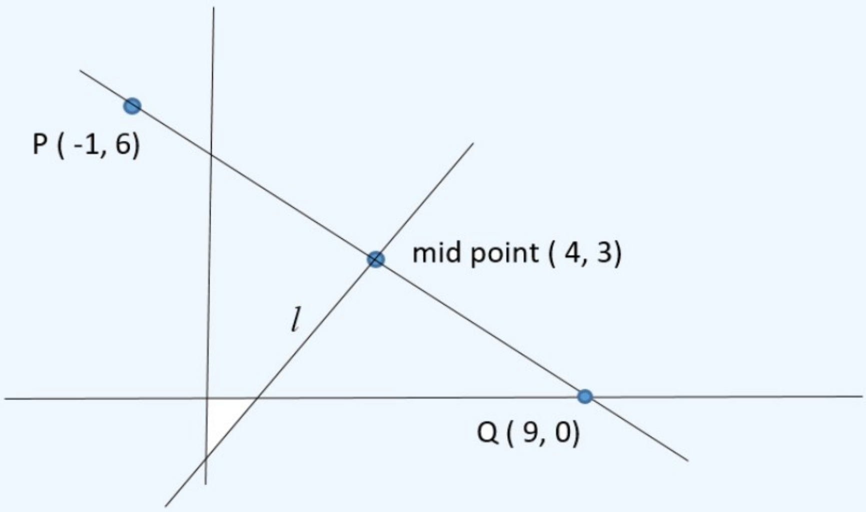
Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Select an answer:

A: $5x + 6y - 7 = 0$ B: $11x - 3y - 9 = 0$

C: $3x + 9y - 15 = 0$ D: $5x - 3y - 11 = 0$

Correct answer: D



The mid point of PQ has coordinates: $(4, 3)$

The line PQ has a gradient given by:

$$\frac{dy}{dx} = \frac{-6}{10} = -\frac{3}{5}$$

Line *l* is perpendicular to line PQ, so:

$$gradient\ l \times -\frac{3}{5} = -1$$

The gradient of line *l* is therefore $\frac{5}{3}$

The line passes through the point x = 4, y = 3. Substituting these values:

$$y = \frac{5}{3}x + c, \qquad so\ \ 3 = \frac{5}{3}(4) + c$$

$$c = \frac{9 - 20}{3} = -\frac{11}{3}$$

Thus, the equation for line *l* is:

$$y = \frac{5}{3}x - \frac{11}{3} \qquad so\ \ 3y = 5x - 11 \qquad or \quad 5x - 3y - 11 = 0$$

8.

The line l_1 has the equation $2x + 3y + 5 = 0$
The line l_2 passes through the coordinates (1, 7) and (5, 1).

Determine, giving full reasons for your answer, whether l_1 and l_2 are parallel, perpendicular or neither.

Correct answer: **neither**

The gradient of line l_2 is given by:

$$\frac{dy}{dx} = \frac{-6}{4} = -\frac{3}{2}$$

The equation for line l_1 can be rearranged:

$$2x + 3y + 5 = 0, \qquad so\ \ 3y = -2x - 5 \qquad or \qquad y = -\frac{2}{3}x - \frac{5}{3}$$

The gradient of line l_1 is therefore $-\frac{2}{3}$

For the lines to be parallel, the gradients must be the same. This is **not** true.

For the lines to be perpendicular, the gradients must multiply to give -1. The gradients actually multiply to give +1, so this is **not** true.

The lines are neither parallel nor perpendicular.

Longer examination questions

1.

The points *A* and *B* have coordinates (–1, 10) and (5, 1) respectively. The straight line *L* has equation $2x - 3y + 6 = 0$.

a) The line *L* intersects the line *AB* at the point *C*. Find the coordinates of *C*. [5]

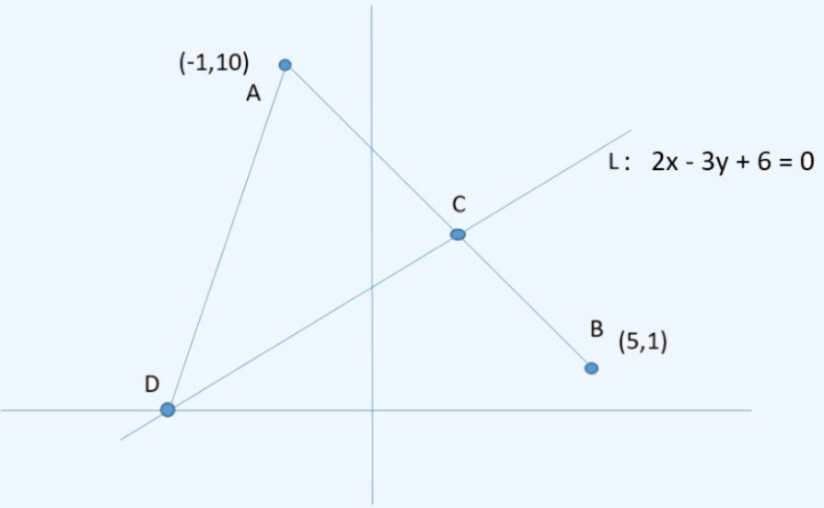
b) Determine the ratio in which the line *L* divides the line *AB*. [2]

c) The line *L* crosses the x-axis at the point *D*. Find the coordinates of *D*. [1]

d) **i)** Show that *L* is perpendicular to *AB*.

ii) Calculate the area of the triangle *ACD*. [6]

The points and lines described in the question can be shown in a diagram:



(a) The line AB has a gradient $\frac{dy}{dx} = \frac{1-10}{5-(-1)} = -\frac{3}{2}$

Line AB passes through the point (5,1). The equation of AB is therefore given by:

$$y = mx + c, \quad so \quad 1 = -\frac{3}{2}(5) + c$$

$$c = 1 + \frac{15}{2} \quad = \quad 8\frac{1}{2}$$

Thus the equation of line AB is: $y = -\frac{3}{2}x + 8\frac{1}{2}$

Line L has the equation $2x - 3y + 6 = 0$, so $3y = 2x + 6$, or $y = \frac{2}{3}x + 2$

At point C, the y-cooordinates on line AB and on line L are equal, so:

$$\frac{2}{3}x + 2 = -\frac{3}{2}x + 8\frac{1}{2}$$

$$\frac{2}{3}x + \frac{3}{2}x = 6\frac{1}{2}$$

$$\frac{4+9}{6}x = \frac{39}{6} \quad so \quad 13x = 39 \quad or \quad x = 3$$

Substituting x = 3 in the equation for line L gives:

$y = \frac{2}{3}(3) + 2$, so y = 4

Point C has the cooordinates (3, 4)

(b) Considering the points A (-1, 10), C (3, 4) and B (5, 1), the line is divided in the ratio 2:1

(c) Line L has the equation: $y = \frac{2}{3}x + 2$

At point D, the y-value is zero:

$$\frac{2}{3}x = -2 \quad so \quad x = -3$$

The line L crosses the x-axis at the point (-3, 0)

(d)

(i) The gradient of line AB is $-\frac{3}{2}$. The gradient of line L is $\frac{2}{3}$. Multiplying these values:

$$-\frac{3}{2}\times\frac{2}{3} = -1$$

which is the required condition for the lines to be perpendicular.

(ii) Since AC and DC are perpendicular, then ACD is a right angled triangle. Its area is given by:

$$base \times \frac{1}{2}\ perpendicular\ height\ =\ AC\ \times \frac{1}{2}\ CD$$

$$AC = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$$

$$CD = \sqrt{4^2+6^2} = \sqrt{52} \qquad \Rightarrow \qquad area = \frac{1}{2}\sqrt{52}\times\sqrt{52}\ = 26$$

2.

The line L_1 passes through the points $A(0, 5)$ and $B(3, -1)$.

a) Find the equation of the line L_1. [3]

The line L_2 is perpendicular to L_1 and passes through the origin O.

b) Write down the equation of L_2. [1]

The lines L_1 and L_2 intersect at the point C.

c) Calculate the area of triangle OAC. [4]

d) Find the equation of the line L_3 which is parallel to L_1 and passes through the point $D(4, 2)$. [2]

(a) The gradient of line L_1 is given by:

$$\frac{dy}{dx} = \frac{-1-5}{3-0} = -2$$

Substituting values of x and y for the point (0, 5):

$$y = mx + c, \quad so \quad 5 = -2(0) + c, \qquad thus \qquad c = 5$$

The equation of line L_1 is: $y = -2x + 5$

(b) The lines L_1 and L_2 are perpendicular, so:

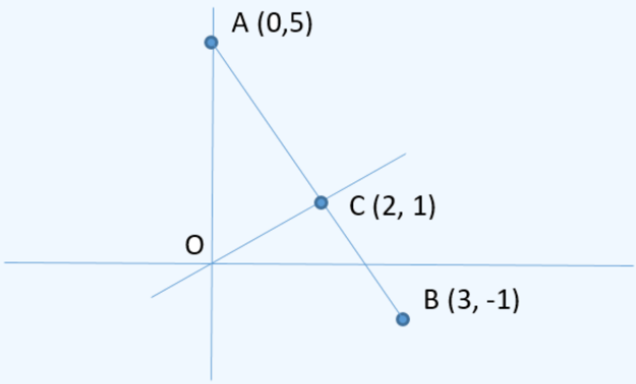
$$gradient\ L_2 \times (-2) = -1, so\ gradient = \frac{1}{2}$$

Substituting values of x and y for the point (0, 0):

$$y = mx + c, \quad so \quad 0 = \frac{1}{2}(0) + c, \qquad thus \qquad c = 0$$

The equation of line L_2 is: $y = \frac{1}{2}x$

(c)



At the point C, the y-coordinates on both lines are equal, so:

$$-2x + 5 = \frac{1}{2}x, \qquad \frac{5}{2}x = 5, \quad thus \quad x = 2$$

Substituting x=2 in the equation for L_1

$$y = -2x + 5 \qquad\qquad y = -2(2) + 5 \qquad thus \;\; y = 1$$

The point C has coordinates (2, 1)

Since the lines meeting at C are perpendicular, OAC is a right angled triangle.

$$area\ OAC = \frac{1}{2}base \times perpendicular\ height$$

$$= \frac{1}{2}\left(\sqrt{4+16}\right) \times \left(\sqrt{4+1}\right) \quad = \quad \frac{1}{2}\left(\sqrt{5}\sqrt{4}\right)\left(\sqrt{5}\right) = 5$$

The area of triangle OAC = 5

(d) Line L_3 is parallel to L_1 so has the same gradient of -2. L_3 passes through the point (4, 2)

$$y = mx + c, \quad so \;\; 2 = -2(4) + c\,, \qquad thus \qquad c = 10$$

The equation of line L_3 is: $y = -2x + 10$

3.

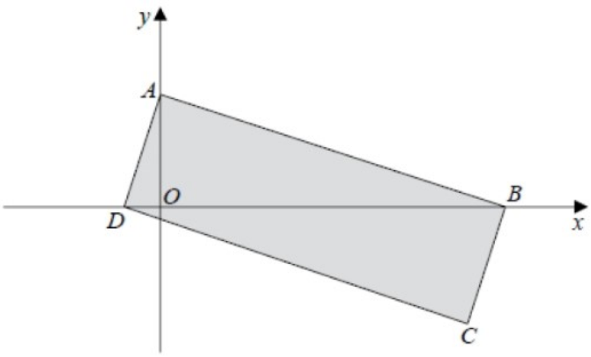


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point A lies on the y-axis and the points *B* and *D* lie on the x-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

The line through points A and B has the formula:

$$5y + 2x = 10, \quad so \quad 5y = -2x + 10$$

$$y = -\frac{2}{5}x + 2$$

The gradient of line AB is therefore $-\frac{2}{5}$.

The lines AB and AD are sides of a rectangle, so are perpendicular. Therefore:

$$gradient\ AB \times gradient\ AD = -1$$

$$gradient\ AD = \frac{-1}{\left(-\frac{2}{5}\right)} = \frac{5}{2}$$

The y-coordinate of point A can be found by substituting x = 0 into the equation for line AB:

$$y = -\frac{2}{5}x + 2, \quad so \quad y = 2$$

Point A has the coordinates (0, 2)

Line AD has the formula: $y = mx + c, \ \ where\ m = \frac{5}{2}$

The line passes through (0, 2) so: $2 = \frac{5}{2}(0) + c$ c = 2

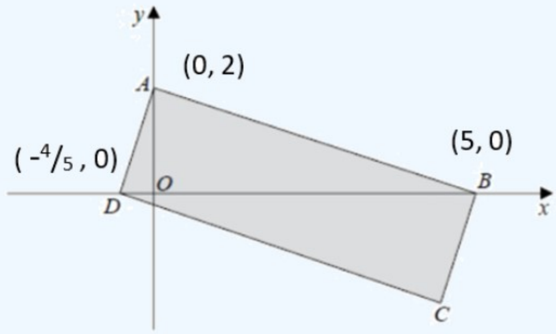
The equation for line AD is $y = \frac{5}{2}x + 2, \quad or \quad 2y - 5x = 4$

(b) Using the equation for line AD, the x-coordinate of point D can be found by setting y = 0:

$$-\frac{5}{2}x = 2, \quad so \quad x = -2 \times \frac{2}{5} \qquad x = -\frac{4}{5}$$

Using the equation for line AB, the x-coordinate of point B can be found by setting y = 0:

$$\frac{2}{5}x = 2, \quad so \quad 2x = 10 \qquad x = 5$$



Length of the rectangle = $\sqrt{5^2 + 2^2}$ Width of the rectangle = $\sqrt{2^2 + \left(\frac{4}{5}\right)^2}$

Area of rectangle = $\sqrt{5^2 + 2^2} \times \sqrt{2^2 + \left(\frac{4}{5}\right)^2} = 11.6$

4.

The line L_1 passes through the points $A(-1, 3)$ and $B(2, 9)$. The line L_2 has equation $2y + x = 25$ and intersects L_1 at the point C. L_2 also intersects the x-axis at the point D.

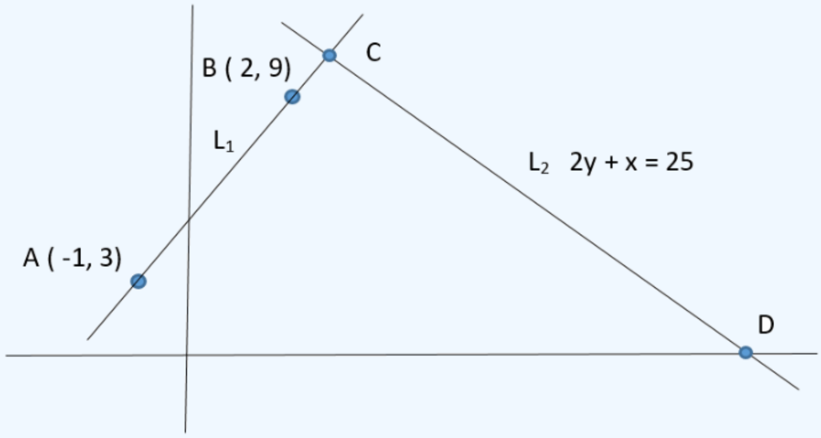
a) Show that the equation of the line L_1 is $y = 2x + 5$. [3]

b) **i)** Find the coordinates of the point D.

ii) Show that L_1 and L_2 are perpendicular.

iii) Determine the coordinates of C. [5]

Showing the points and lines described in the question:



(a)

Line L_1 passes through the points (-1, 3) and (2, 9). The gradient is given by:

$$gradient\ L_1 = \frac{dy}{dx} = \frac{(9-3)}{(2-(-1))} = \frac{6}{3} = 2$$

Line L_1 passes through the point x = -1, y = 3. Substituting these values:

$$y = 2x + c, \qquad 3 = -2 + c \qquad so \qquad c = 5$$

Thus, the equation of line L_1 is: $y = 2x + 5$

(b)(i) At point D, y = 0. Substituting this value in the equation for L_2 :

$$2y + x = 25, \; so \; x = 25$$

The coordinates of point D are (25, 0)

(ii) The gradient of L_1 is 2.

For line L_2: $2y + x = 25, \; so \; 2y = -x + 25 \qquad y = -\frac{1}{2}x + 12\frac{1}{2}$

The gradient of line L_2 is therefore $-\frac{1}{2}$

$$gradient\ L_1 \times gradient\ L_2 = 2 \times \left(-\frac{1}{2}\right) = -1,$$

This is the required condition for the lines to be perpendicular.

(iii) At point C, the y-coordinates of L_1 and L_2 are equal, so:

$$2x + 5 = -\frac{1}{2}x + 12\frac{1}{2}$$

$$4x + 10 = -x + 25, \qquad so \quad 5x = 15, \qquad x = 3$$

Substituting in the equation for L_1: $y = 2(3) + 5 = 11$

Point C has the coordinates (3, 11)

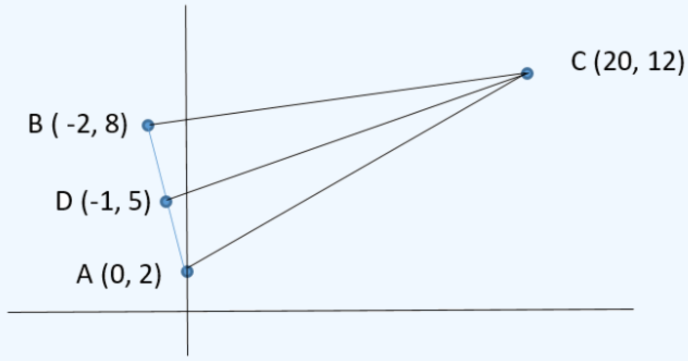
5.

The points *A*(0, 2), *B*(– 2, 8), *C*(20, 12) are the vertices of the triangle *ABC*.
The point *D* is the mid-point of *AB*.

(*a*) Show that *CD* is perpendicular to *AB*. [6]

(*b*) Find the exact value of tan $C\hat{A}B$. [5]

(*c*) Write down the geometrical name for the triangle *ABC*. [1]



(a) The gradient of the line AB is given by:

$$\frac{dy}{dx} = \frac{-6}{2} = -3$$

The mid point D of AB is the point (-1, 5)

The gradient of the line CD is given by:

$$\frac{dy}{dx} = \frac{7}{21} = \frac{1}{3}$$

Multiplying the gradients of the two lines:

$$-3 \times \frac{1}{3} = -1$$

which is the required condition for the lines to be perpendicular.

(b) The angle CÂB is equal to the angle CÂD in the right angled triangle CAD

$$\tan C\hat{A}D = \frac{CD}{AD}$$

$$CD = \sqrt{21^2 + 7^2} \qquad AD = \sqrt{1^2 + 3^2}$$

Therefore:

$$\tan C\hat{A}D = \frac{\sqrt{490}}{\sqrt{10}} = \sqrt{49} = 7$$

(c) Triangle ABC is symmetrical about the line CD, so is an **isosceles** triangle.

6.

ABCD is a trapezium where *A* is the point (1, −2), *B* is the point (7, 1) and *C* is the point (3, 4)

DC is parallel to *AB*.

AD is perpendicular to *AB*.

(a) (i) Find the equation of the line *CD*.

[2 marks]

(a) (ii) Show that point *D* has coordinates (−1, 2)

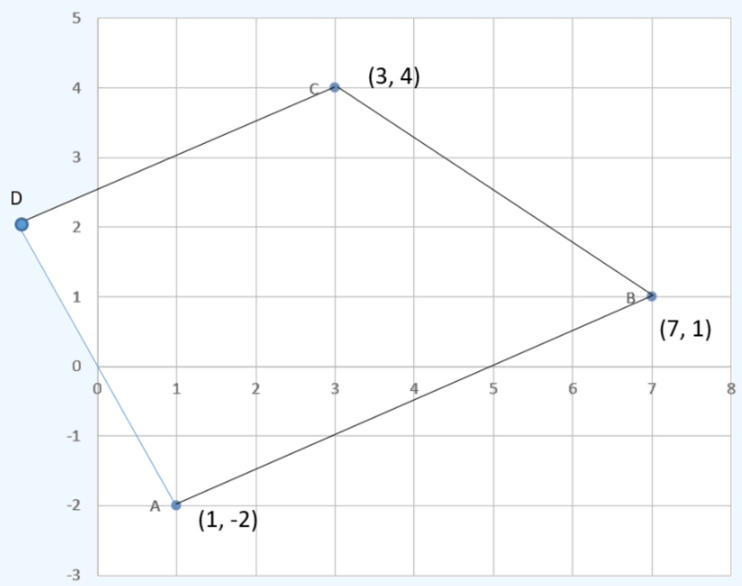
[3 marks]

(b) (i) Find the sum of the length of *AB* and the length of *CD* in simplified surd form.

[2 marks]

(b) (ii) Hence, find the area of the trapezium *ABCD*.

[2 marks]



(a)(i) Line AB has gradient $\frac{dy}{dx} = \frac{3}{6} = \frac{1}{2}$

Line CD is parallel to AB, so has the same gradient.

CD passes through the point (3, 4). Substituting:

$$y = mx + c \quad gives \quad 4 = \frac{1}{2}(3) + c, \quad so \ \ c = \frac{5}{2}$$

The equation of line CD is: $y = \frac{1}{2}x + \frac{5}{2}$

(ii) AD is perpendicular to AB, so its gradient is -2 . This ensures that the product of the gradients of the perpendicular lines is -1

Line AD passes through the point (1, -2). Substituting these values:

$$y = mx + c \quad gives \quad -2 = -2(1) + c, \quad so \ \ c = 0$$

Thus, the equation of line AD is: $y = -2x$

At point D, the y cooordinates of the two lines AD and AC are equal, so:

$$\frac{1}{2}x + \frac{5}{2} = -2x, \quad so \quad x + 5 = -4x, \quad x = -1$$

Substituting in the equation for line AD, $y = -2(-1) = 2$

The coordinates of point D are (-1, 2)

(b)(i) Length AB = $\sqrt{6^2+3^2} = \sqrt{45} = \sqrt{5}\sqrt{9}$ Length CD = $\sqrt{4^2+2^2} = \sqrt{20} = \sqrt{5}\sqrt{4}$

$$AB + CD = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$$

The area of the trapezium = given by: $\frac{AB+CD}{2} \times AD$

Length AD = $\sqrt{4^2+2^2} = \sqrt{20} = \sqrt{5}\sqrt{4}$

Thus, the area is: $\frac{5\sqrt{5}}{2} \times 2\sqrt{5} = 25$

7.

The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.
(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.
(2)

Given that O is the origin,

(c) find the area of the triangle OAB.
(2)

(a) The gradient of line l_2 must be $\frac{1}{2}$, so that the product of the gradients of the two perpendicular lines = -1

Line l_2 passes through the point x = 5, y = 6. Substituting these values in the equation:

$$y = mx + c \qquad so \qquad 6 = \frac{1}{2}(5) + c \qquad c = \frac{12-5}{2} = \frac{7}{2}$$

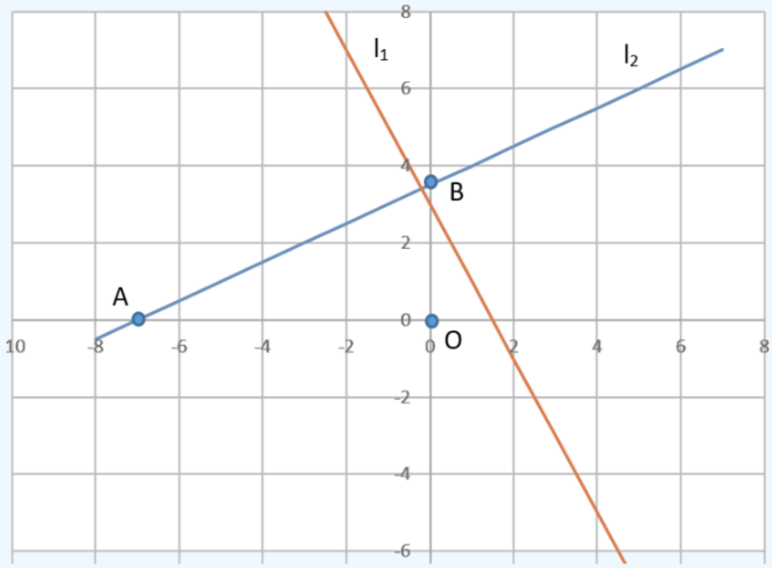
Thus, the equation for l_2 is: $y = \frac{1}{2}x + \frac{7}{2}$ $\quad or \quad$ $x - 2y + 7 = 0$

(b) At point A, y = 0, so $x + 7 = 0$ $\qquad x = -7$

The coordinates of point A are (-7, 0)

At point B, x = 0, so $-2y + 7 = 0$ $\quad 2y = 7$ $\quad y = 3\frac{1}{2}$

The coordinates of point B are (0, $3^1/_2$)



The area of triangle OAB =

$$\frac{1}{2}\, base \times perpendicular\ height \quad = \quad \frac{1}{2}\left(7 \times 3\frac{1}{2}\right) \quad = \quad 12.25$$

8.

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A $(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(a) The line L_1 passes through the point A where x = 1, y = 4.
Substituting these values in the equation of the line:

$$2(4) - 3(1) - k = 0 \qquad so\ 8 - 3 = k \qquad k = 5$$

(b) Rearranging the equation:

$$2y - 3x - 5 = 0, \quad so \quad 2y = 3x + 5 \qquad y = \frac{3}{2}x + \frac{5}{2}$$

The gradient of the line L_1 is $\frac{3}{2}$

(c) Line L2 is perpendicular to L1, so its gradient must be $-\frac{2}{3}$

$$gradient\ L_1 \times gradient\ L_2 = \frac{3}{2} \times -\frac{2}{3} = -1$$

which is the required condition for the lines to be perpendicular.

Substituting the coordinates of point A (1,4) in the equation for line L_2:

$$4 = -\frac{2}{3}(1) + c, \qquad so\ c = 4\frac{2}{3}$$

$$y = -\frac{2}{3}x + \frac{14}{3}, \quad so \quad 3y = -2x + 14$$

$$2x + 3y - 14 = 0$$

(d) Where L_2 crosses the x-axis, y=0. Substituting in the equation of L_2:

$$2x - 14 = 0, \quad so \quad x = 7$$

Poiny B has the coordinates (7, 0)

(e) Line AB extends from point (1, 4) to point (7, 0)

The length of the line is given by Pythagoras' theorem:

$$AB = \sqrt{6^2 + 4^2} = \sqrt{52} = \sqrt{4}\sqrt{13} = 2\sqrt{13}$$
