2 Student numeracy

There is agreement between leaders in politics, business and education that a high level of numeracy in the workforce is important to the future prosperity of Britain. A National Numeracy Programme has been developed for Wales (Welsh Assembly Government, 2012), emphasising that numeracy skills should be taught throughout schools and colleges, across all subjects. The wider definition of numeracy, encompassing skills such as problem solving and communication, is accepted:

> Numeracy is an essential life skill and learners need to be able to apply this skill across the curriculum in different subject areas, and in real life contexts. Mathematics is a part of numeracy, but to be numerate means you are able to apply some of these mathematical skills in many more contexts than in mathematics lessons and across several subject areas. It is therefore our expectation that all teachers will be teachers of numeracy.

During our own investigation, senior lecturing staff of our college were asked for their opinions on the problems of delivering student numeracy within vocational courses. Integration of numeracy was seen as a desirable objective, but could be prevented by poor student mathematical ability:

> It makes far more sense to teach numeracy, and literacy if it comes to that, through the subject, as students can see the relevance to their real life work. Having said that, there may be a problem in that the standard of maths is so poor that if they haven't grasped the basics then they cannot cope. You may have to take them out, unfortunately, to get those basics. But in an ideal world, applying numeracy to engineering or carpentry or whatever course they are doing is far more meaningful.

Let us examine the structure of mathematics education. School mathematics in Britain, as in many other countries, is designed around a bottom-up academic model. Pupils learn mathematical methods within distinct topic areas such as: number, algebra and geometry, then work on example applications still within these same topic areas. The intention is that pupils will progress to study subjects at an advanced level, such as sciences, where they will be able to make good use of the mathematical techniques they have learned.

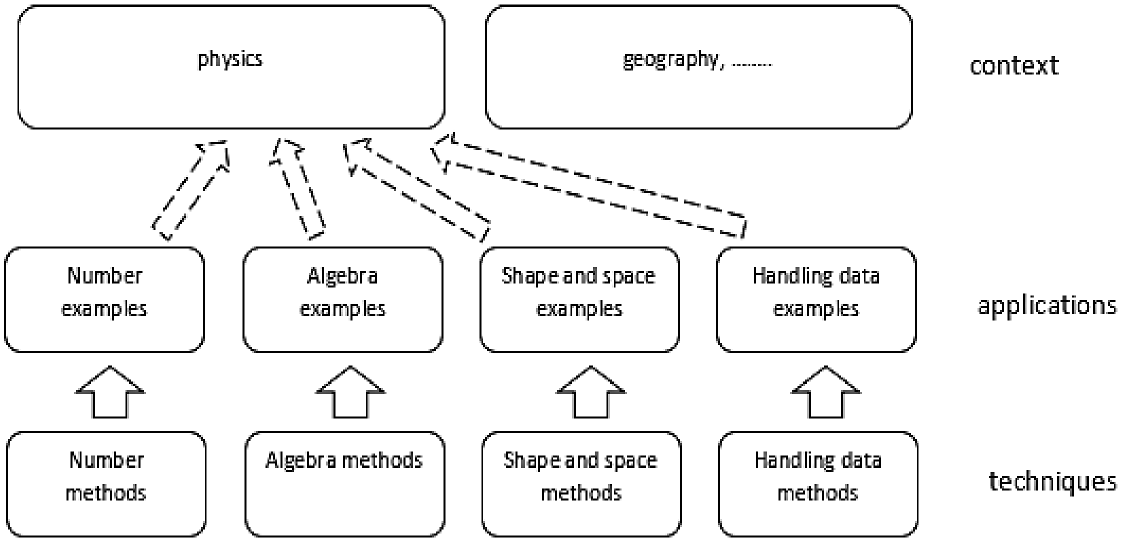

Figure 3: Bottom-up academic mathematics model

This model can present problems for students who leave school at the age of 16 to study a practical vocational course. They may view mathematics as a series of unrelated topics, some of which seem to have no relevance to their chosen profession. Algebra, in particular, is seen by many school leavers as having very little practical everyday use.

As an alternative to making mathematical methods the starting point, the approach we have taken is to work downwards within the vocational area to identify numeracy tasks that practitioners might need to undertake in their everyday work. The tasks are then analysed by the students and solved using mathematical methods, which might be familiar or which might need to be learned at this point. Additionally, the work provides opportunities for consolidating mathematical knowledge in these broader topic areas.

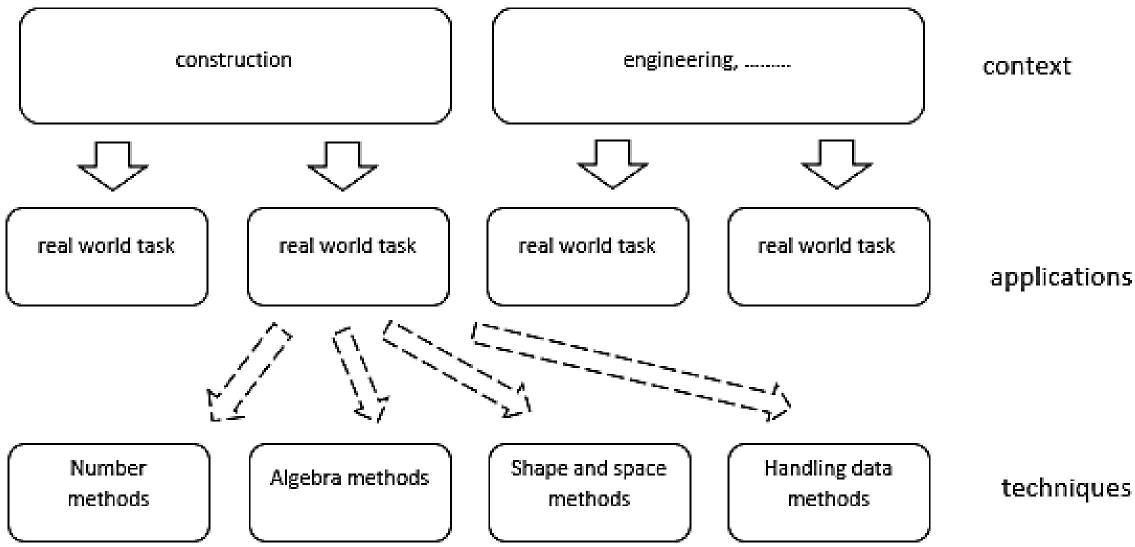

Figure 4: Top-down vocational numeracy model

Research authors disagree about the extent to which numeracy can be taught as a transferable skill outside of the workplace (Figure 5). For example: it is difficult to directly relate the skill of operating a Computer Numeric Controlled lathe to the topics in school mathematics. There are, however, a variety of transferrable numeracy skills involved, for example: geometry for understanding tool paths, algebra for program commands, and use of number for checking measurements.

Basic skills **Utilitarian**	**Insulationist** **Situated cognition**
Eraut, Cockcroft	Lave
⟵	⟶
mathematical concepts are abstract and transferrable between contexts, e.g. **finding percentages**	mathematical concepts are deeply embedded in the context, e.g. **using a CNC lathe**

Figure 5: Mathematical concepts in the workplace

To capitalise on the two approaches, we might teach specific vocational topics in numeracy as an opportunity to present mathematical concepts that could be generalised to other applications. This would allow students to develop their problem solving skills in non-routine situations

After outlining the possible overall structure that numeracy provision might take in colleges, let us now turn our attention to what is perhaps the central issue in developing numeracy: the attitude of the student towards the subject. Without a positive attitude, learning is unlikely to be effective. Beswick, Watson & Brown (2006) comment that:

> Attitude to mathematics has been recognised as a multi-dimensional construct that includes confidence or anxiety; liking or disliking mathematics; an inclination to engage in or to avoid mathematics; beliefs about whether one is good or bad at mathematics; and beliefs that mathematics is important or unimportant, useful or useless, easy or difficult, and interesting or uninteresting.

Each successive *Trends in International Mathematics and Science Study* (Mullis et al., 2012) has shown a strong positive relationship between students' attitudes toward mathematics and their mathematics achievement.

Students often begin vocational courses with a poor experience of school mathematics and lack enthusiasm to improve their mathematical skills (Lewis, 2013, 2014).

> Boredom was identified as the single most common negative emotion….Pupils in top sets for mathematics exhibit dissatisfaction and aspects of disaffection with school mathematics as much as pupils in lower sets.

According to the Organisation for Economic Cooperation and Development PISA Survey of student attitudes (OECD, 2012):

> In the United Kingdom, students do not report high levels of intrinsic motivation. Only 56% of students agreed or strongly agreed that they are interested in learning mathematics: 59% of boys and 53% of girls.

A differentiation in attitude related to gender is identified:

> Girls in the United Kingdom do not enjoy mathematics, are anxious when asked to solve mathematical problems, and underperform compared with boys.

Beswick, Watson & Brown (2006) comment on the poor motivation of some students for mathematics:

> Students who do not see mathematics as being integral to their academic self-concept try to avoid the anxiety resulting from involvement in mathematical tasks. As they are unlikely to have or to develop the motivation to learn mathematics, the teacher should be patient, encouraging, and supportive of students' individual learning styles

Middleton and Spinias (1999) observe that motivation towards mathematics is developed early, is highly stable over time, and influenced greatly by teacher actions and attitudes. They comment that:

> Students need a relatively high degree of success in mathematics for engagement in mathematics to be perceived as worthwhile, and they need to feel that success in mathematics is attributable to their ability and effort.

Lewis (2013) provides a case study of a student, Anna, who did well in mathematics at primary school, but lost her motivation for the subject when transferred to a low achievers' group in secondary school:

> Anna’s story contains a dominant narrative, and this is one of her anger and anguish at being classified as a “C/D” and her corresponding performance at that “failed” level. This experience is in stark contrast to her early, innocent years, before labelling and failure, when she was able to get on with school mathematics in a seemingly untroubled way. However, the consequences of her failure go deep, and provide the motivational and emotional context in which all of her subsequent experiences are filtered and interpreted.

Dweck (1986) introduced the concept of *learned helplessness*. This refers to a state where the student has decided that they are unable to succeed due to lack of ability, and see no purpose in continuing to struggle with the subject.

Fennema et al. (1990) raise the issue that teachers may inadvertently reinforce learned helplessness for mathematics in girls through differential treatment of the genders. Boys who experience difficulties may be perceived as showing a lack of effort, whilst girls may be perceived as having a lack of ability.

We were pleased to find from a survey in our own college that students generally have a positive attitude towards maths and numeracy, not seeing it as a problematic aspect of their studies, and getting a sense of satisfaction from successful mathematical problem solving. They are willing to persevere if they make mistakes.

Mathematics seems to be viewed by our students as a necessary tool in problem solving which has its difficulties on occasions, but that these can be overcome with effort. There is a fairly neutral view of mathematics, as neither particularly enjoyable nor particularly unpleasant. There seems little interest in experimenting with mathematics beyond what is necessary to solve practical problems in the students' vocational areas or everyday life.

It is perhaps unsurprising that our students have reached college with the view that mathematics has a very rigid structure, with right and wrong answers and correct ways of solving particular problems. There seems to be a perception that formulae have to be learned in order to solve particular classes of problems. This is consistent with the findings of Ketterlin-Geller (2007), who comments that:

> By the time algebra is introduced in middle school, many students view mathematical principles as subjective and arbitrary and rely on memorization in lieu of conceptual understanding.

Our students seem confident in their use of everyday mathematics and do not find the mathematics within their courses threatening. They are starting to be aware of the importance of properly understanding the mathematical methods that they are using. Mathematics is seen as becoming more interesting at higher levels, but there is little desire to go further in studying mathematics as a specialist subject in its own right. We would agree with the recommendation of the Advisory Committee on Mathematics Education:

> A wider curriculum and provision than exists at present should be developed in order to ensure that all young people are well placed to benefit from their studies in mathematics. While it is very desirable to increase the number of students taking AS and A-levels in mathematics and further mathematics, these students will only ever be in a minority and attention must also be directed to the rest of the cohort.

However, here we perceive a problem. Numeracy activities included as an add-on to the main timetable are often not valued by students, who understandably wish to focus on their main vocational course. Students may feel that their time is being wasted if techniques already learned at GCSE level are repeated. Even if a student was unsuccessful in school mathematics and requires remedial help, it is unlikely that the student will find a repeat of the same material motivating.

The Essential Skills Numeracy course was not seen as useful by many of the students whom we interviewed:

> 'Although I can see the reasons behind Essential Skills Numeracy, doing science courses means that I am working with numbers and it is of little value to me. The requirements for it are very basic, so I find it very dull. My suggestion for improving it would be to integrate it with students' courses, so that they could complete the work along with their other subjects as well as learn new maths which is useful for those subjects.'
>
> 'There have been times when I was made to take subjects that were clearly intended to improve numeracy skills that plainly didn't work. The Application of Number course for the Welsh Bac was so simplistic and boring that I ended up learning nothing.'
>
> 'The course did not really teach us how to measure anything or how to make decisions on how to check the data. We were spoon fed data and told exactly what to do with it. We were even given pre-prepared results for any tables we needed.'
>
> 'Application of Number wasn't much help to my learning. We were told to pick a destination for a trip we had to organise, but we were then given an enormous specification that limited the choices severely. This produced a sheer lack of interest.'

Essential Skills Numeracy was perceived as too simple to be of educational value or interest, and a need for differentiation was identified by several students.

There is no doubt that a genuine need exists to improve numeracy, due to the difficulties experienced by some students when faced with numeracy problems.

During our project, clinical interviews (Ginsburg, 1981) were carried out with students chosen from the range of courses. The students were asked to give a commentary on their reasoning whilst attempting to solve various mathematical problems. From an analysis of the interview transcripts, four particular difficulties were identified:

- Lack of specialised mathematical vocabulary. Students had difficulty describing features of graphs, equations and other mathematical entities.
- No strong connection between number and algebra in problem solving (Lee & Wheeler, 1989). Students made no attempt to understand relationships in formulae by substituting numerical values, and made no attempt to devise formulae to simplify the repetitive handling of numerical data.
- A preference for justification by concrete example. Students generally preferred to use physical measurement of solid shapes to solve problems, rather than abstract mathematical reasoning.
- Misuse of standard algorithms which had been learned in a superficial manner without full understanding. Examples causing difficulty included formulae for areas and volumes, lengths of sides of triangles, and trigonometry.

Tasks involving algebraic techniques were generally perceived as more difficult than tasks involving shape or number. Some differences in perception of difficulty exist between the different vocational groups that we surveyed, which might be related to practical numeracy tasks which the students undertake during their courses. Carpentry students seem more confident with geometric problems, whilst business students are happier with algebraic tasks.

Numeracy difficulties are by no means confined to Further Education, and problems are also experienced in Higher Education. The report "Every Student Counts" published by the Higher Education Academy (Tariq et al., 2010) quotes a psychology lecturer:

> Because teaching time is limited, and because staff time is very pressurised, I don't feel that we have enough time to really explain the background to statistics (i.e. how everything is derived) in a way that students find meaningful. They get the theory in their lectures, but I don't think they find it easy to tie that in with the practical elements (e.g. use of SPSS), and by the time they're getting to grips with the practical stuff, the theory is just a distant memory.

Motivation appears central to students' successful development of numeracy. Middleton and Spinias (1999) see it as much better to develop intrinsic motivation through the value and interest of the subject material and tasks presented, rather than to provide extrinsic motivation through rewards or penalties based on performance:

> To facilitate the development of students' intrinsic motivation, teachers must teach knowledge and skills that are worth learning. Students must understand that the mathematics instruction they receive is useful, both in immediate terms and in preparing them to learn more in areas in which mathematics can be applied, such as: physics or business. Use of ill-structured, real-life problem situations in which the use of mathematics facilitates uncovering important and interesting knowledge promotes this understanding.

Our survey responses demonstrate the value of allowing students to choose, design and take ownership of tasks, as a means of increasing motivation.

> The main application of number during my time in the college has been in practical work which has revolved around topics I have chosen, thus I have had an interest in them. This means that I have been more driven to research and think through the problems.

Our students were asked for their opinions on numeracy activities which they had undertaken in the college as part of their courses. Students enthusiastically described numeracy tasks in which they had been involved and which they clearly found worthwhile and interesting:

> ‘During physics classes we were told to create contraptions to demonstrate different physical laws. I made a crane to prove the principle of moments. I had to add a counterweight on the opposite end to the main weight to maintain equilibrium.’

> 'For computing I am producing a program which is a solar system simulation. I contacted Aberystwyth University, who gave me a table of data on the geometry of the orbits of the planets.'
>
> 'In business, we are doing trial balances and balance sheets. We are also working on financial gearing ratios which I find interesting and will help me in the future.'
>
> 'We had to take part in an enterprise challenge, where we had to sell products to make a profit. We had to calculate the budget to buy equipment and ingredients. I felt that this challenge has improved my numeracy.'
>
> 'I carried out a task planning for a new kitchen, a new bathroom, modernising the flat, and repairs to the roof. I estimated the costs and planned the work to do in the workshop and on site. This helped me with my arithmetic, percentages and estimation, and will make work in my trade a lot easier.'

A clear preference was shown for practical numeracy. This may be equated with components of the extended adult learning model: participation in a community of practice, and reflection on practice:

> 'In physics we have practical sessions. These are fun. We get experience in setting up electronic measuring equipment and see where the data is coming from, and we understand how errors can happen. We have to use the appropriate formulae and sometimes modify these to get the right answers.'
>
> 'During electronics lessons we conduct experiments, usually involving multimeters to test voltages and resistances in circuits under different circumstances. These experiments nearly always involve numeracy. We would take the data and create a graph to show the relationships, or use formulas to analyse the data. I find this a very effective and enjoyable way of working using numeracy.'
>
> 'Computing lessons often require us to solve a mathematical problem with a program, but we are not give any clues on how to solve it as they are all different. Therefore we have to identify the numbers and try to create a formula to give the correct answer. This is fun, as it allows us to think and find ways to solve a problem that we understand – because we have created the solution method ourselves.'
>
> 'I find maths easy in the workshop and it's interesting and useful.'

Our students expressed less interest in numeracy tasks which were not presented in a practical vocational context:

> 'In work that isn't practical, I feel that the numeracy is not as interesting. This is because I enjoy collecting data in the first place and feel more inclined to work with it.'
>
> 'Often we are given example examination questions but they don't really teach us anything. These are often boring as we have been studying the topics for long periods of the course and are then asked to work through the same things over and over.'

> 'Exams and traditional theory work give the students the impression that they are being fed information as opposed to learning it. To get them engaged, you have to get them involved with collecting the data itself.'

We find the MeE motivational model of Munns and Martin (2005) to be a useful framework for the development of numeracy courses. The model focuses on the need to present students with interesting learning activities and the satisfaction of achievement from engaging successfully with these activities. This can in turn lead to students developing a personal engagement with the subject as a whole, through the enjoyment and interest which it provides, so that they become intrinsically motivated to develop their skills and knowledge to a higher level:

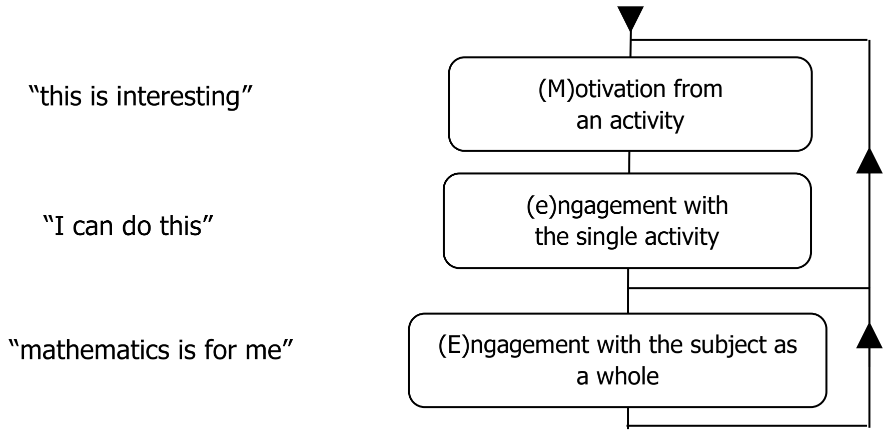

Figure 6: MeE motivation model of Munns and Martin (2005)

Munns and Martin advocate the introduction of the most interesting work from the very start of a course, as a means of generating enthusiasm. Teachers may need to simplify tasks to ensure that students reach a successful outcome and gain a sense of achievement. It is important that students consider the tasks to be realistic, relevant and worthwhile.

A number of authors and organisations have developed motivating approaches to numeracy education. We might mention:

- Work by the Nuffield Foundation in developing the AS-level Use of Mathematics qualification and associated teaching resources. This acknowledges the importance of allowing students autonomy in planning and carrying out their own mathematical investigations as a means of developing problem solving skills, effective independent learning skills, and in improving motivation. Hernandez-Martinez et al. (2011) have made a study of the project component of AS-level Use of Mathematics, and identified in students: a perceived greater depth of understanding; increased motivation and learning through mathematical modelling and use of technology; changes to learning activities which promoted student-centred learning; and assessment that better suited some students.

- Work in China by groups of further education teachers, notably: Tang, Sui and Wang (2003). This work has identified a range of strategies for incorporating realistic problem solving activities into vocational courses, and has again demonstrated the value of this approach in improving student motivation and mathematical skills.
- Work in Singapore (Ang, 2001) and Australia (Stillman et al., 2007) on the use of mathematical modelling as a means of developing students' problem solving abilities in real world situations.

These examples of innovative teaching approaches in numeracy have a number of features in common: problems presented to students are realistic real-world situations; students must analyse the problems and devise their own methods of solution; data must be obtained by measurement or literature search; the data must be processed, often with the help of computer technology; and finally the problem solution must be presented in a form which is clear to the intended audience.

The approaches outlined above are consistent with both the needs of employers, and the requirements for motivating students to improve their numeracy skills.

Many authors emphasise the importance of the teacher. Beswick, Watson & Brown (2006) make the comment:

> Students will feel more comfortable taking risks if they know that they will not be criticized or humiliated for making mistakes.

Tutor support can be vital, particularly where students are experiencing difficulties and feel insecure. Carl Rogers (2001) identifies three characteristics of supportive tutors:

- acceptance and liking of the student as an individual, irrespective of any problems they might be causing
- empathic understanding of the difficulties that the student might be facing
- honesty in interactions with the student so that a mutual trust is developed.

Students will not feel comfortable to ask for help from a tutor whom they sense does not like them, has no understanding of their problems, or is likely to conceal information from them or be unreliable in the arrangements they make on their behalf.

Several studies have been carried out of teachers' attitudes to mathematics and numeracy (Beswick, Watson & Brown, 2006). It is clear that there is a need for a higher level of mathematical knowledge in order to develop confidence to create innovative activities.

Where learning activities are successful, distinctions may need to be made between the inherent content of the activity, the manner in which it was presented by the teacher, and the social environment in which the activity was undertaken. These three factors may be equated to the theory of Jaworski (1992), who has proposed a triad of components for effective mathematics teaching (Fig. 7).

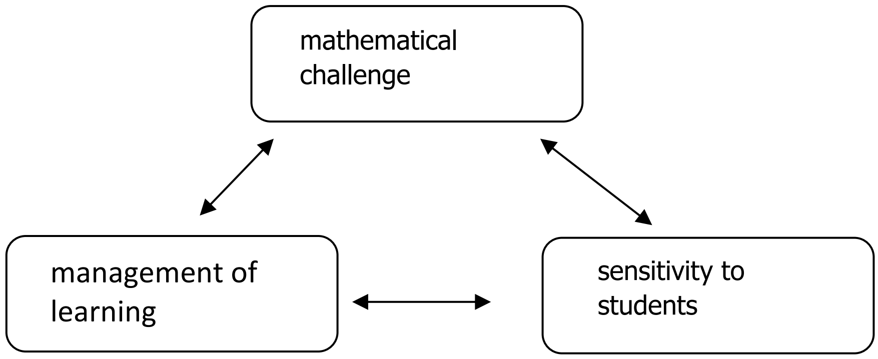

Figure 7: Mathematics teaching model of Jaworski

Mathematical challenge is achieved by providing students with a stimulating mathematical activity which they see as worthwhile.

Management of learning relates to the manner in which the teacher organises the activity, perhaps allowing whole group discussions or feedback by individual students on the progress of the task.

Sensitivity to students allows the teacher to recognise opportunities for presenting new mathematical methods and skills through pedagogic learning

Of the vocational groups we surveyed, computing students appear most confident in handling mathematical tasks, which might be related to the strong mathematical basis of the subject through the use of data variables and graphics coordinates. However, a significant minority of the computing group see numeracy as difficult. Lecturers in computing, and in many other subjects, are likely to be teaching groups with very mixed numeracy ability. It may be important to consider the design of learning activities which allow for differentiation, in order not to dishearten weaker students or lose the interest of the more able students.

Students are motivated to study through different goals (Meece, Blumenfeld and Hoyle, 1988).

A *mastery* goal orientation occurs when a student develops intrinsic motivation to achieve a particular level of skill or knowledge.

A *competitive* goal orientation describes a student's motivation as based on out-performing other members of the student group. However, a competitive goal is seen as significantly less effective in achieving learning outcomes then a mastery goal.

A *social* goal orientation represents a desire by the student to participate in studies because they enjoy being with friends in the class. Provided that a social goal is combined with a genuine interest in studying the subject, this can provide strong motivation to participate in course activities.

Examination of previous research indicates that motivation is a major factor in the development of student numeracy. Perhaps more than in any other subject, students can develop negative attitudes towards mathematics through doubting their own ability. Negative attitudes can be countered by a combination of supportive teaching, the presentation of interesting learning activities, and the motivating effects of successful outcomes. Tutors may need to be particularly aware of the need to sensitively support girls who have developed a state of learned helplessness in mathematics.

The approach taken in developing student motivation may be an important factor for success in numeracy. A mastery goal in which the student takes pleasure in achieving a satisfactory standard is seen as more desirable than a competitive goal in which students merely wish to out-perform their peers. This implies that more emphasis should be placed on gaining the skills to independently undertake real-world numeracy tasks, with students gaining a sense of achievement through successful problem solving. Less emphasis should be placed on routine standardised assessments based on specified mathematical techniques.

A further factor which can make a mastery goal more desirable than a competitive goal is in developing students' ability to work cooperatively with others. Effective group working is seen by employers as an important factor amongst staff, but involves complex social as well as work skills. Underwood (2003) discusses social interaction problems which can arise within student groups undertaking project tasks. Success is more likely when students are working towards a shared objective than when competing with one another.

Summary

Good student motivation seems to be the key to developing numeracy skills during vocational courses. We have found that motivation is highest when numeracy activities are integrated into course activities in a way that realistically simulates problem solving in the workplace.

Students become most engaged when presented with practical projects where they can collect their own data and have the autonomy to develop their own solutions. This approach can help in the development of problem solving skills, and encourages a deep understanding of the numeracy techniques which are selected and used in the project.

We might discuss the effective integration of numeracy activities into vocational courses in terms of the Jaworski teaching model:

> **Mathematical challenge** requires the selection of a genuinely interesting and realistic problem or project related to the student's vocational area of study.
>
> **Management of learning** might allow students to work in small groups to plan a solution strategy, collect the necessary data, carry out the analysis, and report on the results.
>
> **Sensitivity to students** would involve the tutor taking on the role of an advisor and consultant. The tutor would unobtrusively monitor progress and offer advice and specialist information about suitable mathematical techniques. A classroom climate would be encouraged in which mastery and social goals are promoted.

Supervising open-ended project work can present a challenge. Tutors will benefit from a broad background knowledge of both the vocational area and the relevant mathematical techniques available for problem solving.